OCTOBER, 1960
No. 4
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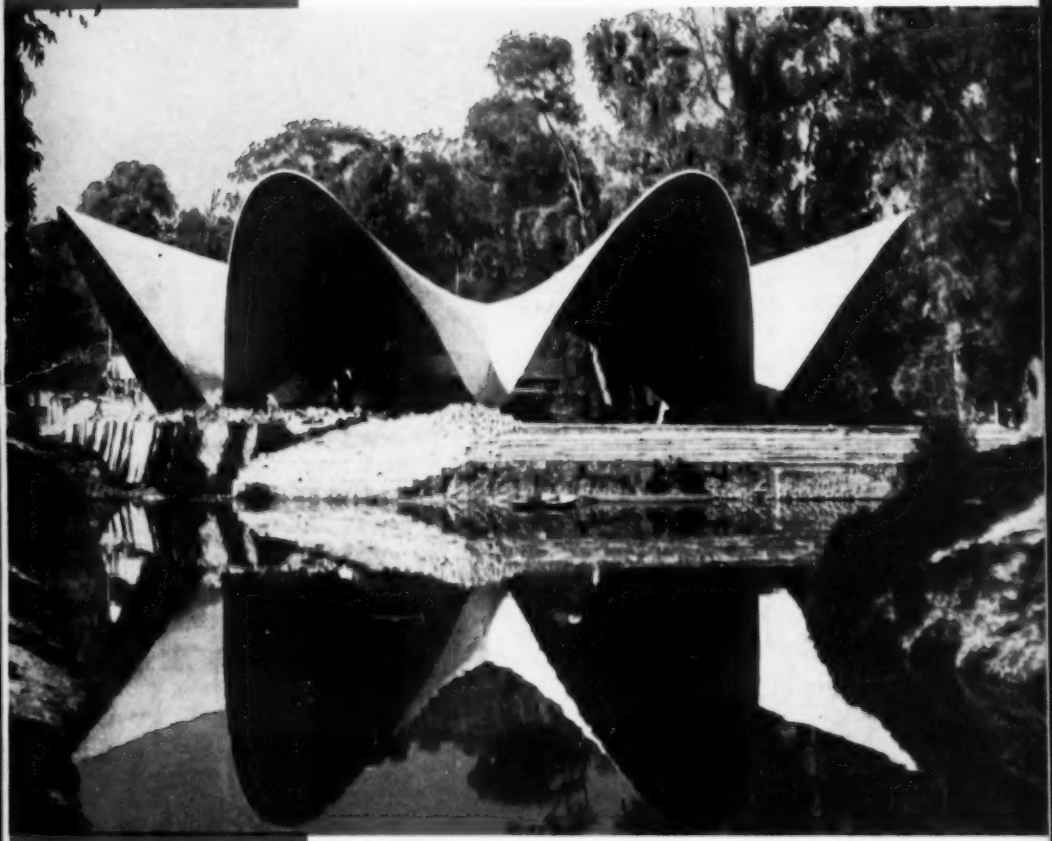

ACI
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V. 57

Shells, hyperbolic and elliptical paraboloids, are featured this month in five papers beginning on p. 353

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Proceedings V. 57

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Coming next month in the JOURNAL

BORIS BRESLER opens the November issue with a discussion of "Design Criteria for Reinforced Columns Under Axial Load and Biaxial Bending."

Field and laboratory observations of "Corrosion of Prestressed Wire in Concrete" constitute the basis of the research paper presented by G. E. MONFORE and G. J. VERBECK.

A. M. NEVILLE and J. TAUB offer Part 4 of a five-part series on "The Resistance to Shear of Reinforced Concrete Beams. Part 4—The Behavior of Beams with Different Types of Web Reinforcement.

L. H. KRISTOF discusses the development and use of "Monolithic Cast-in-Place Concrete Pipe" from its inception to the present, including the merits of this type of water carrying conduit. General summary is made of advantages and limitations of different construction methods.

LYLE E. YOUNG describes a method of "Simplifying Ultimate Flexural Theory by Maximizing the Moment of the Stress Block," without dependence on experimental beam tests.

K. M. ALEXANDER reports on extensive research on the "Reactivity of Ultrafine Powders Produced from Siliceous Rock."

On the cover—Restaurant "Los Manantiales," Xochimilco, Mexico, is roofed with a groined concrete vault on an octogonal plan formed by the intersection of four hyperbolic paraboloids 1-1/2 in. thick. Outside diameter is 140 ft. Architect was Joaquin Alvarez Ordonez. Structure: Felix Candela, Cubiertas ALA, S. A. This graceful structure is a fitting symbol for the theme of this month's JOURNAL.

Manuscripts of papers, discussions, and reports should be sent in triplicate to:

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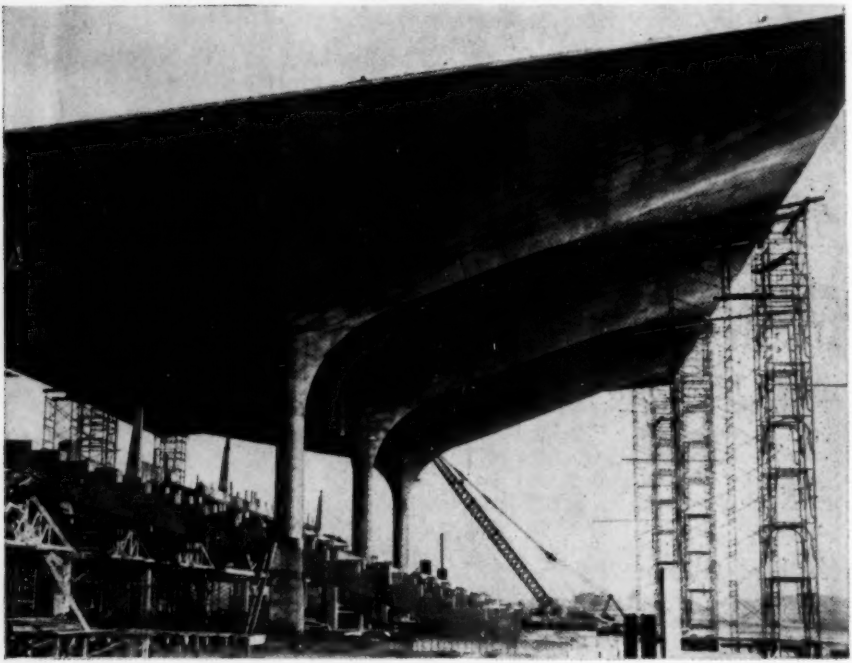

PLASTIMENT speeds placement of thin shell concrete

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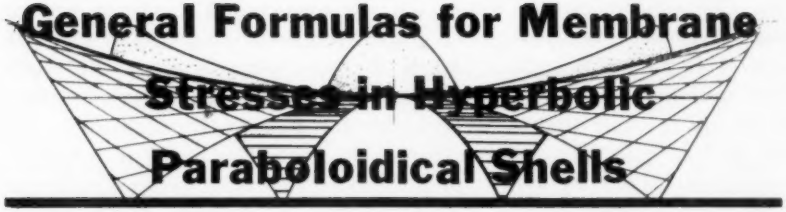
Title No. 57-16

General Formulas for Membrane Stresses in Hyperbolic Paraboloidical Shells



By FELIX CANDELA

Complementing a previous paper,[15] a set of general formulas are presented for reinforced hyperbolic paraboloidical shells. These are those giving the membrane stresses in a shell loaded with its own weight but disposed in an arbitrary position in space. The better known formulas for a hyperbolic paraboloidical shell with its z axis vertical can be obtained as a special case of the general formulas. A numerical example of a groined vault on a square plan is also presented.

■ SINCE 1955, WHEN A PREVIOUS PAPER by the author appeared on these pages,[15] the "hypar"* has received the favor of architects and engineers and is now widely used the world around.

Just after the publication of that paper the author became aware of several errors in the text, which make obscure the exposition. Also the final formulas were unnecessarily complicated by a change of variables.

Instead of just making the strict corrections on the original text, it has been considered convenient, in view of the increasing employment of this surface in reinforced concrete shells, to publish a more general set of formulas. These are those giving the membrane stresses in a hypar loaded with its own weight but disposed in an arbitrary position in space, that is, when the load has components along the three axes of the hypar. The better known formulas for the hypar with its z axis in a vertical position can be obtained as particular cases of this more general set of equations.

Notation and figures are the same as in the previous paper, except when noted.

*To simplify matters, it has been suggested that the name hyperbolic paraboloid be shortened to hypar. This contraction will be used here.

SURFACE DEFINITION

Assume two straight nonparallel, nonintersecting lines, *HOD* and *ABC* (Fig. 1), in space, which will be provisionally named *directrix*. Straight lines h_n that intersect both directrix, being at the same time parallel to one plane xOz named *director plane,* define the surface. They will be called the *first system of generators.* The two directrix determine in their turn a second director plane yOz, parallel to them. The surface may also be considered as created by a *second system of generators* i_n parallel to this plane and intersecting every generator h_n of the first system.

The hypar contains, therefore, two systems of straight lines h_n and i_n, each system being parallel to a director plane and both planes forming an arbitrary angle ω. Every point of the surface is the intersection of two straight lines contained in the surface.

Taking as coordinate axes the two generators passing by the crown of the hypar and the hypar axis or intersection of both director planes, the equation of the surface, in these birectangular coordinates, will be:

$$z = kxy \qquad (1)$$

k being a constant which represents the *unitary slope* or *warping* of the hypar [in Fig. 1, $k = AA'/(OB \cdot OH)$]; $xOy = \omega$ can be any angle; xOz and yOz are right angles.

This is the *simplest possible equation* of second degree tying together the three coordinates of each point. When the director planes form a

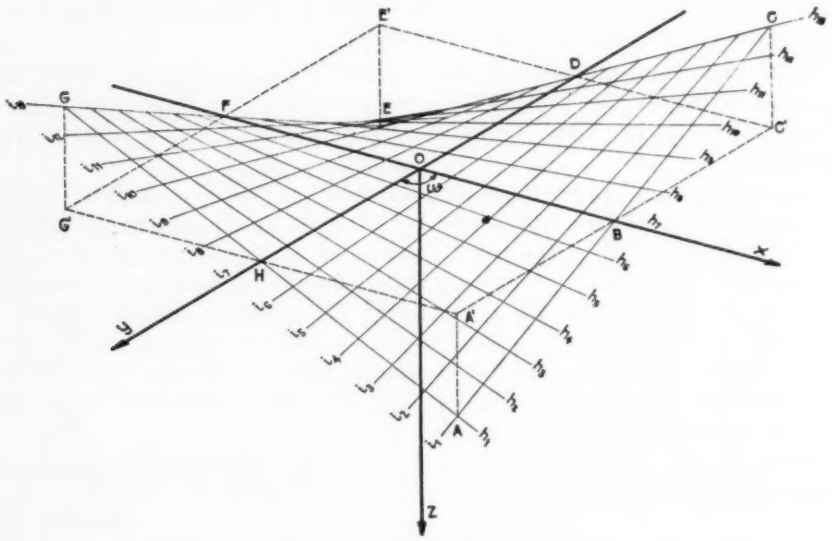


Fig. 1—Surface definitions

Felix Candela, architect, Mexico City, Mexico, is well known for his work in the field of concrete shell structures. Mr. Candela is the founder of Cubiertas ALA, S.A., which specializes in such structures, and Aeroblock, S. de R. L., which produces foamed concrete. An ACI member since 1947, he is currently a member of Committee 334, Concrete Shell Structures.

right angle ($\omega = 90$ deg) the hypar is *equilateral* or *rectangular*. When ω is any other angle, the hypar is *oblique*. Plane sections parallel to the bisecting planes of the director dihedral angle xOy are parabolic. They are named *principal parabolas*, and are respectively curved upward (GOC) and downward (AOE); hence the surface is *anticlastic* or inversely doubly-curved. All other plane sections and their projections on plane xy are hyperbolas or their degeneration in two straight lines, except those parallel to the z axis which are parabolas and, of course, those parallel to the director planes which give the straight generators.

As a translation surface (Fig. 2), the hypar may be considered as generated by a principal parabola ABC that moves parallel to itself along an inverse principal parabola BOF. Therefore the surface has two systems of parabolic generators. Each system is composed of identical parabolas situated in parallel planes.

STRESS EQUATIONS IN BIRECTANGULAR AXES

Equations of membrane stresses* in a surface represented by $z = f(xy)$, in a system of coordinate axes in which angle xOy can have any value ω and xOz, yOz are right angles, are obtained[6] by expressing the equilibrium along x, y, and z of the forces acting on the surface element shown in Fig. 3, neglecting the differentials of second order. They are:

$$\frac{\delta v_x}{\delta x} + \frac{\delta \tau}{\delta y} = -X \sin \omega \qquad (2a)$$

*It must be understood that stresses here are, strictly speaking, stresses times the shell thickness.

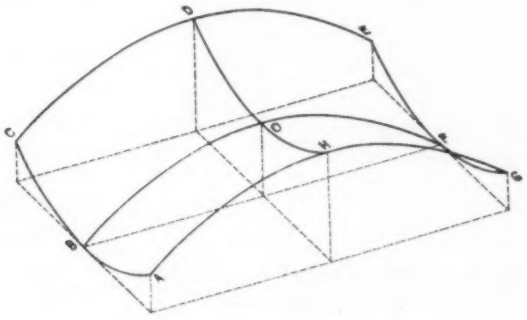


Fig. 2—The paraboloid surface has two systems of parabolic generatrix

$$\frac{\delta \tau}{\delta x} + \frac{\delta v_y}{\delta y} = -Y \sin \omega \quad \text{(2b)}$$

$$r v_x + t v_y + 2s\tau = (pX + qY - Z) \sin \omega \quad \text{(2c)}$$

where, according to Monge's notation

$$p = \frac{\delta z}{\delta x}\ ;\ q = \frac{\delta z}{\delta y}\ ;\ r = \frac{\delta^2 z}{\delta x^2}\ ;\ s = \frac{\delta^2 z}{\delta x\, \delta y}\ ;\ t = \frac{\delta^2 z}{\delta y^2} \quad \text{(3)}$$

and v_x, v_y and τ are the projections of real stresses on plane xy (Fig. 3)

$$v_x = \sigma_x \sqrt{\frac{1 + q^2}{1 + p^2}}\ ;\quad v_y = \sigma_y \sqrt{\frac{1 + p^2}{1 + q^2}}\ ;\quad \tau = T \quad \text{(4)}$$

and X, Y and Z are the coordinate components of external forces measured by unit of surface projection on plane xy. All forces, stresses, and directions of coordinate axes represented in Fig. 3 are taken as positive.

Since the hypar is represented by Eq. (1)

$$p = ky,\quad q = kx,\quad r = t = 0,\quad s = k \quad \text{(5)}$$

and Eq. (2c) becomes

$$2k\tau = (kyX + kxY - Z) \sin \omega \quad \text{(6)}$$

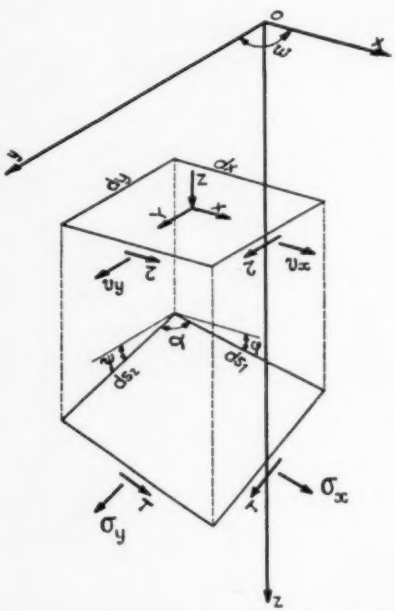


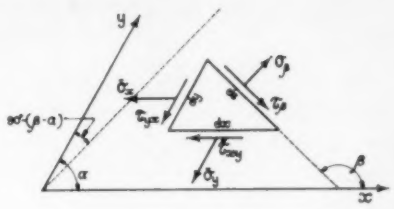
Fig. 3—Angle α is formed by two intersecting generatrix

Differentiating Eq. (6) with respect to x and y, substituting in Eq. (2a) and (2b), and integrating the resulting expressions, the general solutions for v_x and v_y are obtained, containing respectively arbitrary functions of either y or x which must be determined by the boundary conditions. Eq. (4) gives the final values of real stresses σ_x, σ_y, and τ.

It should be noted that σ_x and σ_y are oblique components of stresses, since they are parallel to the sides ds_1 and ds_2 of the element of surface. Therefore, the usual constructions of Mohr's or dyadic circles cannot be used to obtain the stresses in other sections, except approximately in flat rectangular hypars.

Angle α (Fig. 3) formed by two intersecting generators is obtained

Fig. 4—Conditions of equilibrium



by the following formula

$$\cos \alpha = \frac{pq + \cos \omega}{\sqrt{(1 + p^2)(1 + q^2)}} \quad \ldots\ldots (7)$$

Analogously, angle β formed by any section contained in the first quadrant with the positive part of the x generator passing by the point (Fig. 4) is obtained by

$$\cos \beta = \frac{pq_\beta + \cos \omega_\beta}{\sqrt{(1 + p^2)(1 + q_\beta{}^2)}} \quad \ldots\ldots (8)$$

in which ω_β is the projection of angle β on the plane xy and q_β is the trigonometrical tangent of the angle formed by the straight line representing the section with the xy plane. To avoid mistakes, a strict sign convention should be observed. The sign of product pq_β will be positive when both sides of the angle ω are ascending or descending to the vertex. It will be negative when one side is going up and the other going down to the vertex.

Normal and tangential stresses σ_β and τ_β on any such section are obtained by consideration of the equilibrium in the σ_β and τ_β directions, respectively, in Fig. 4.

$$\sigma_\beta = \sigma_x \frac{\sin^2 \beta}{\sin \alpha} + 2\tau \frac{\sin\beta \sin(\beta - \alpha)}{\sin \alpha} + \sigma_y \frac{\sin^2(\beta - \alpha)}{\sin \alpha} \quad \ldots\ldots (9a)$$

$$\tau_\beta = -\sigma_x \frac{\sin \beta \cos \beta}{\sin \alpha} - \tau \frac{\sin(2\beta - \alpha)}{\sin \alpha} - \sigma_y \frac{\sin(\beta - \alpha)\cos(\beta - \alpha)}{\sin \alpha} \quad \ldots\ldots (9b)$$

Directions of principal stresses are determined by setting τ_β equal to zero in Eq. (9b)

$$\tan 2\theta = \frac{2\tau \sin\alpha + \sigma_y \sin 2\alpha}{\sigma_x + 2\tau \cos\alpha + \sigma_y \cos 2\alpha} \quad \ldots\ldots (10)$$

where θ and $(90 - \theta)$ are the angles of principal stresses with the positive part of the x generator passing by the point. Values of principal stresses are:

$$\sigma_I = \sigma_x \frac{\sin^2\theta}{\sin\alpha} + 2\tau \frac{\sin\theta \sin(\theta - \alpha)}{\sin\alpha} + \sigma_y \frac{\sin^2(\theta - \alpha)}{\sin\alpha} \quad \ldots\ldots (11a)$$

$$\sigma_{II} = \sigma_x \frac{\cos^2\theta}{\sin\alpha} + 2\tau \frac{\cos\theta \cos(\theta - \alpha)}{\sin\alpha} + \sigma_y \frac{\cos^2(\theta - \alpha)}{\sin\alpha} \quad \ldots\ldots (11b)$$

HYPAR WITH UNIFORMLY DISTRIBUTED SURFACE LOAD (DEAD LOAD)

We will assume in this analysis that the z axis of the hypar has an arbitrary position in space. Hence the load will have three components X_1, Y_1 and Z_1, along the three hypar axes. These components are related to the forces of X, Y, and Z appearing in Eq. (2a), (2b), and (2c) by the relation between the real area of the surface element and the area of its projection on plane xy.

$$X\,dx\,dy\sin\omega = X_1\,ds_1\,ds_2\sin\alpha \qquad (12)$$

From Eq. (7) we obtain:

$$\sin^2\alpha = \frac{1 + p^2 + q^2 + p^2q^2 - p^2q^2 - \cos^2\omega - 2pq\cos\omega}{(1 + p^2)\,(1 + q^2)}$$

$$\sin\alpha = \frac{\sqrt{\sin^2\omega + p^2 + q^2 - 2pq\cos\omega}}{\sqrt{(1 + p^2)\,(1 + q^2)}} \qquad (13)$$

besides (Fig. 3)

$$\frac{ds_1}{dx} = \sec\varphi = \sqrt{1 + p^2} \qquad (14a)$$

$$\frac{ds_2}{dy} = \sec\psi = \sqrt{1 + q^2} \qquad (14b)$$

Substituting Eq. (13), (14a), and (14b) in Eq. (12),

$$X\sin\omega = X_1\sqrt{\sin^2\omega + p^2 + q^2 - 2pq\cos\omega} = X_1\sqrt{\phi} \qquad (15a)$$

where

$$\phi = \sin^2\omega + p^2 + q^2 - 2pq\cos\omega = \sin^2\omega + k^2y^2 + k^2x^2 - 2k^2xy\cos\omega \qquad (16)$$

Analogously

$$Y\sin\omega = Y_1\sqrt{\phi} \qquad (15b)$$

$$Z\sin\omega = Z_1\sqrt{\phi} \qquad (15c)$$

and finally Eq. (2a), (2b), and (2c) transform into

$$\frac{\delta v_x}{\delta x} + \frac{\delta\tau}{\delta y} = -X_1\sqrt{\phi} \qquad (17a)$$

$$\frac{\delta\tau}{\delta x} + \frac{\delta v_y}{\delta y} = -Y_1\sqrt{\phi} \qquad (17b)$$

$$\tau = \left(\frac{y}{2}\,X_1 + \frac{x}{2}\,Y_1 - \frac{Z_1}{2k}\right)\sqrt{\phi} \qquad (17c)$$

Eq. (17c) is an algebraic expression which gives univocally the value of τ at each point. Differentiating (17c) with respect to x and y, substituting these differentials in Eq. (17a) and (17b), and integrating the

resulting expressions with respect to y and x, the equations for v_x and v_y are obtained.

$$\frac{\delta\tau}{\delta y} = \tfrac{1}{2}X_1\sqrt{\phi} = k^2X_1\frac{y(y - x\cos\omega)}{2\sqrt{\phi}} + k^2Y_1\frac{x(y - x\cos\omega)}{2\sqrt{\phi}}$$

$$- kZ_1\frac{y - x\cos\omega}{2\sqrt{\phi}}$$

substituting in (17a)

$$v_x = -\tfrac{1}{2}\int\Big[3X_1\sqrt{\phi} + \frac{k^2X_1y^2 - kZ_1y}{\sqrt{\phi}} + \frac{(k^2Y_1y - k^2\cos\omega X_1y + k\cos\omega Z_1)x}{\sqrt{\phi}}$$

$$- k^2\cos\omega Y_1\frac{x^2}{\sqrt{\phi}}\Big]dx$$

solving the above integral yields

$$v_x = \Big[\tfrac{1}{4}(\cos\omega Y_1 - 3X_1)x + \Big(\frac{5}{4}\cos\omega X_1 - \tfrac{1}{2}Y_1 + \frac{3}{4}\cos^2\omega Y_1\Big)y$$

$$- \frac{1}{2k}\cos\omega Z_1\Big]\sqrt{\phi} + \Big[\Big(\frac{5}{5}k\sin\omega X_1 - \frac{3}{4}k\cos\omega \sin^2\omega Y_1\Big)y^2$$

$$+ (\tfrac{1}{2}Z_1\sin^2\omega)y - \frac{1}{4k}\sin^2\omega(3X_1 + \cos\omega Y_1\Big]\mathrm{Log}_n\frac{kx - ky\cos\omega + \sqrt{\phi}}{\sin\omega\sqrt{1 + k^2y^2}} + f_1(y)$$

or in abridged form

$$v_x = (A_1x + A_2y + A_3)\sqrt{\phi} + (A_4y^2 + A_5y + A_6)\,\mathrm{Log}_n(X_7) + f_1(y) \quad (18a)$$

where

$$A_1 = \frac{1}{4}(\cos\omega Y_1 - 3X_1)$$

$$A_2 = \frac{5}{4}\cos\omega X_1 - \frac{1}{2}Y_1 + \frac{3}{4}\cos^2\omega Y_1$$

$$A_3 = -\frac{1}{2k}\cos\omega Z_1$$

$$A_4 = -\frac{5}{4}k\sin^2\omega X_1 - \frac{3}{4}k\cos\omega\sin^2\omega Y_1$$

$$A_5 = \frac{1}{2}\sin^2\omega Z_1$$

$$A_6 = -\frac{1}{4k}\sin^2\omega(3X_1 + \cos\omega Y_1)$$

$$X_7 = \frac{kx - ky\cos\omega + \sqrt{\phi}}{\sin\omega\sqrt{1 + k^2y^2}}$$

analogously

$$v_y = (B_1y + B_2x + B_3)\sqrt{\phi} + (B_4x^2 + B_5x + B_6)\,\mathrm{Log}_n(Y_7) + f_2(x) \quad (18b)$$

where

$$B_1 = \tfrac{1}{4}(\cos\omega\, X_1 - 3Y_1)$$

$$B_2 = \frac{5}{4}\cos\omega\, Y_1 - \tfrac{1}{2}X_1 + \frac{3}{4}\cos^2\omega\, X_1$$

$$B_3 = -\frac{1}{2k}\cos\omega\, Z_1 = A_3$$

$$B_4 = -\frac{5}{4}\, k\sin^2\omega\, Y_1 - \frac{3}{4}\, k\cos\omega\sin^2\omega\, X_1$$

$$B_5 = \tfrac{1}{2}\sin^2\omega\, Z_1 = A_5$$

$$B_6 = -\frac{1}{4k}\sin^2\omega\,(3Y_1 + \cos\omega\, X_1)$$

$$Y_7 = \frac{ky - kx\cos\omega + \sqrt{\phi}}{\sin\omega\sqrt{1 + k^2x^2}}$$

$f_1(y)$ and $f_2(x)$ in Eq. (18a) and (18b) are arbitrary functions of integration which allow us to satisfy certain edge conditions.

PARTICULAR CASES

Simplifications of Eq. (17c), (18a), and (18b) for other positions of the axes with respect to the vertical or for rectangular hypars are easily obtained by annulling the corresponding values in the general expressions. The most common simplifications are:

(a) $\omega = 90$ deg (Rectangular hypar)

$$\sin\omega = 1,\quad \cos\omega = 0,\quad A_3 = B_3 = 0\quad \sqrt{\phi} = \sqrt{1 + k^2y^2 + k^2x^2}$$

$$\tau = \left(\frac{1}{2}\, X_1y + \frac{1}{2}\, Y_1x - \frac{1}{2k}Z_1\right)\sqrt{\phi} \qquad (19a)$$

$$v_x = \left(-\frac{3}{4}\, X_1x - \frac{1}{2}\, Y_1y\right)\sqrt{\phi}$$

$$+ \left(-\frac{5}{4}\, kX_1y^2 + \frac{1}{2}\, Z_1y - \frac{3}{4k}X_1\right)\mathrm{Log}_e\frac{kx + \sqrt{\phi}}{\sqrt{1 + k^2y^2}} + f_1(y) \qquad (19b)$$

$$v_y = \left(-\frac{3}{4}\, Y_1y - \frac{1}{2}\, X_1x\right)\sqrt{\phi}$$

$$+ \left(-\frac{5}{4}\, kY_1x^2 + \frac{1}{2}\, Z_1y - \frac{3}{4k}Y_1\right)\mathrm{Log}_e\frac{ky + \sqrt{\phi}}{\sqrt{1 + k^2x^2}} + f_2(x) \qquad (19c)$$

(b) $X_1 = 0$, $Y_1 = 0$ (Hypar with the z axis vertical) (Fig. 5)

$$\tau = \frac{1}{2k}\, Z_1\sqrt{\phi} \qquad (20a)$$

$$v_z = -\frac{1}{2k}\, Z_1\cos\omega\sqrt{\phi} + \frac{1}{2}\sin^2\omega\, Z_1y\,\mathrm{Log}_e\frac{kx - ky\cos\omega + \sqrt{\phi}}{\sin\omega\sqrt{1 + k^2y^2}} + f_1(y) \qquad (20b)$$

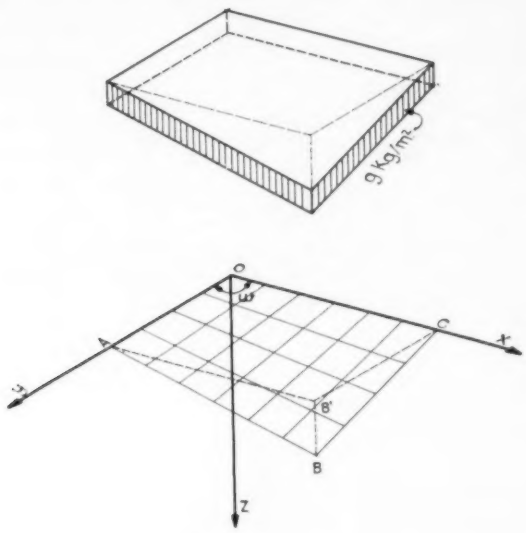


Fig. 5 — Uniformly distributed surface load in z direction — z axis vertical

$$v_y = -\frac{1}{2k} Z_1 \cos\omega \sqrt{\phi} + \frac{1}{2} \sin^2\omega Z_1 x \mathrm{Log}_n \frac{ky - kx\cos\omega + \sqrt{\phi}}{\sin\omega\sqrt{1 + k^2x^2}} + f_2(x) \quad (20c)$$

(c) $X_1 = 0$, $Y_1 = 0$, $\omega = 90$ deg (rectangular hypar with the z axis vertical)

$$\tau = -\frac{1}{2k} Z_1 \sqrt{\phi} \quad (21a)$$

$$v_x = \frac{1}{2} Z_1 y \mathrm{Log}_n \frac{kx + \sqrt{\phi}}{\sqrt{1 + k^2y^2}} + f_1(y) \quad (21b)$$

$$v_y = \frac{1}{2} Z_1 x \mathrm{Log}_n \frac{ky + \sqrt{\phi}}{\sqrt{1 + k^2x^2}} + f_2(x) \quad (21c)$$

(d) $X_1 = 0$, $Y_1 = 0$, $Z_1 = Z = g$ = constant, (Fig. 6). (Uniformly distributed load on horizontal projection, with the z axis vertical)

$$\tau = -\frac{g \sin\omega}{2k} = \text{constant} \quad (22a)$$

$$v_x = f_1(y) \quad (22b)$$

$$v_y = f_2(x) \quad (22c)$$

$f_1(y)$ and $f_2(x)$ being arbitrary functions of integration can have any value, including zero, which gives $v_x = 0$, $v_y = 0$. In this case $\theta = \omega/2$ and both principal stresses have the same absolute value as τ and are directed approximately along the bisecting lines of angle ω.

$$\sigma_I = -\sigma_{II} = |\tau| \quad (23)$$

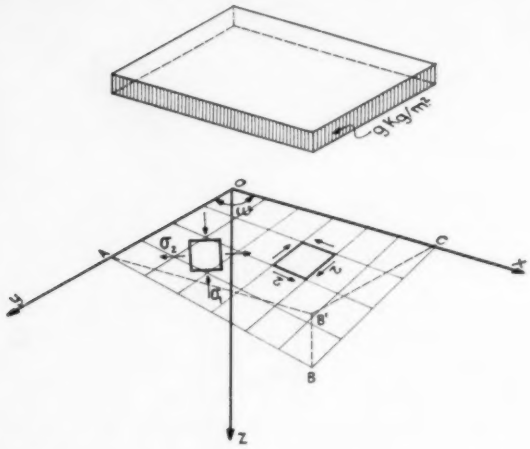


Fig. 6 — Uniformly distributed load on horizontal projection—z axis vertical

This special case is the most commonly known but we must remember that—being an approximate simplification of the general formula, that assumes g practically constant and α almost equal to ω at every point of the surface—it can only be applied in a limited number of cases, where the z axis is vertical and the surface is flat enough to let us consider the real load as uniformly distributed on plane xy. As soon as the rise is increased substantially, or the z axis is not vertical, the more general formulas must be employed.

It must be noted that in all cases the value of τ is definitely fixed by Eq. (17c), but values of ν_x, ν_y and σ_x, σ_y can be variable depending on the choice of values for the arbitrary functions of integration $f_1(y)$ and $f_2(x)$ that appear in Eq. (18a) and (18b). This means that oblique stresses in an unlimited surface are statically indeterminate or hyperstatic. To freeze them we must recur to the boundary or edge conditions. This property gives us some freedom in the choice of the boundary or support disposition.

BOUNDARY CONDITIONS

Hypar limited by straight generators

By giving convenient values to the arbitrary functions of integration it is possible to leave two contiguous sides of any warped quadrangle free of oblique stresses. But of course the stresses at the opposite two sides will be frozen or determinate, and these edges will be subjected to oblique forces which must be absorbed by providing a continuous support along them. This means that in practice it suffices to find the numerical value which must be added to the stresses along each generator (or the lattice of them which we may want to consider) to comply with the support conditions at a certain edge. We may, for instance,

cancel the oblique stress component along one edge, introducing, as Δv_x or Δv_y, a set of stresses equal and opposite to those resulting at this edge from Eq. (18a) or (18b), but this involves the introduction of the same additional stresses at the opposite edge, as if each generator were a tie or a strut, and it will produce, of course, alterations in the state of stress at the interior points of the surface.

From these considerations it becomes clear that a single warped quadrangle, under this type of load, cannot be in equilibrium, except if at least two contiguous sides have edge members or support elements able to resist loads of any direction. Considerations of symmetry in associations of several warped quadrangles may lead to simplifications of the necessary support conditions, but there will always remain nonequilibrated stress components along certain edges (Fig. 7).

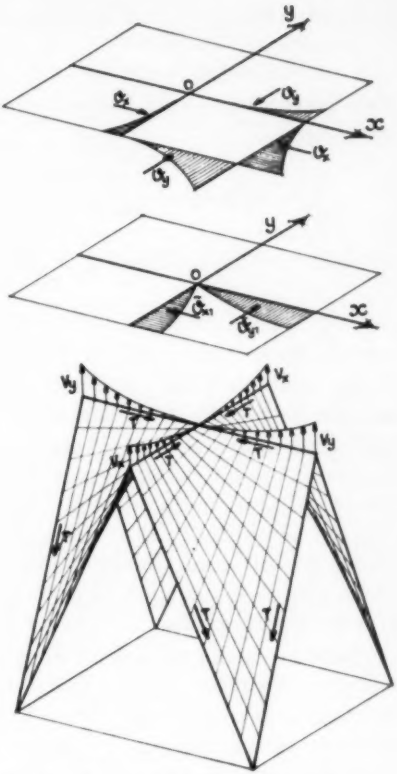


Fig. 7—Symmetrically associated warped quadrangles. Simplification of necessary support conditions may be obtained by considering symmetry

Since the shear stresses along the generators are frozen, it is impossible to leave free of shear stresses any straight edge. The tangential forces resulting from the addition of the shear stresses along the edge must be taken by this edge working in tension or compression.

Hypar with arbitrary curved boundaries

Since in Eq. (9a) and (9b), for the normal and tangential stresses on any section not parallel to the generators, σ_x and σ_y can have any arbitrary value, that is they are variables, it is possible in the case of a curved edge to give any optional value to σ_β and τ_β, including zero. When σ_β and τ_β are annulled along one such edge, it is clear that this edge, being free of stresses, does not require any stiffening member. This can result in an extremely graceful edge. The values of σ_x and σ_y to annul σ_β and τ_β are obtained by setting Eq. (9a) and (9b) equal to zero:

$$\bar{\sigma}_\alpha = -\tau \frac{\sin(\beta - \alpha)}{\sin \beta} \qquad (24a)$$

$$\bar{\sigma}_y = -\tau \frac{\sin \beta}{\sin (\beta - \alpha)} \quad \text{(24b)}$$

If on the other hand we fix beforehand the values of σ_β and τ_β at each point of the edge, the values of σ_x and σ_y to equilibrate those stresses are given by:

$$\bar{\sigma}_x = \sigma_\beta \frac{\cos (\beta - \alpha)}{\sin \beta} - (\tau_\beta + \tau) \frac{\sin (\beta - \alpha)}{\sin \beta} \quad \text{(25a)}$$

$$\bar{\sigma}_y = (\tau_\beta - \tau) \frac{\sin \beta}{\sin (\beta - \alpha)} - \sigma_\beta \frac{\cos \beta}{\sin (\beta - \alpha)} \quad \text{(25b)}$$

Once the values of $\bar{\sigma}_x$ and $\bar{\sigma}_y$ at the boundary points have been determined, their values along the generators intersecting this edge are frozen and no longer indeterminate. When these generators intersect another edge, the resultant edge stresses [σ_β and τ_β obtained by Eq. (9a) and (9b)] must be taken care of by a complete support, i.e., a support able to resist forces in any direction. Symmetrical arrangements of several hypars may lead to simplifications of the necessary support conditions. For instance, at the groins of any symmetrical groined vault there remain only forces contained in the plane of the groin, since forces normal to this plane cancel each other with those proceeding from the contiguous surface.

There are some important considerations with regard to the practical analysis. We must bear in mind, for instance, that when we investigate the points of any lattice formed by generators, we are using a method of finite differences in which the real length of each interval changes with its position, but the stresses acting at each point of intersection are unit stresses or stresses by unit of length. When translating stresses from one to the other end of a generator, the difference of length of the interval on which these stresses are acting at each end must be taken into account.

On the other hand, if we use the projections of the stresses ν_x and ν_y and the projections of the intervals on plane xy we are dealing with a uniform grid, in which the intervals are constant. To translate the stresses we must first find ν_x from σ_x at one end, translate ν_x to the other end, and then find σ_x from ν_x at the latter end.

NUMERICAL EXAMPLE

Groined vault on square plan formed by the intersection of two hypars with vertical axes (Fig. 8).

Data: Side of the square, $L = 20$ m
Height of the crown, $H = 10$ m
Thickness = 4 cm

$$\frac{\omega}{2} = \tan^{-1} \frac{5}{10} = 26 \text{ deg } 34 \text{ minutes}, \quad \omega = 53 \text{ deg } 8 \text{ minutes}$$

Since the plan of the structure is a square, our task is much simpler and we need only to investigate the Triangle 1-6-16-1 (i.e., an eighth of the structure). The basic steps are:

1. Find the coordinates of the boundary points 1 to 30 of the grid of generators we have plotted in the surface, after having determined the numerical value of the parameter k of the hypar (Table 1, Columns 1, 2, 3).

$$k = \frac{Z_{16}}{X_{16}\ Y_{16}} = \frac{10}{16{,}77051 \times (-5{,}59017)} = -0.106667$$

2. Find the basic value of τ, v_z, v_y at the points 1 to 30, from Eq. (20a), (20b), and (20c), (Case B). See Table 1, Columns 4, 5, 6.

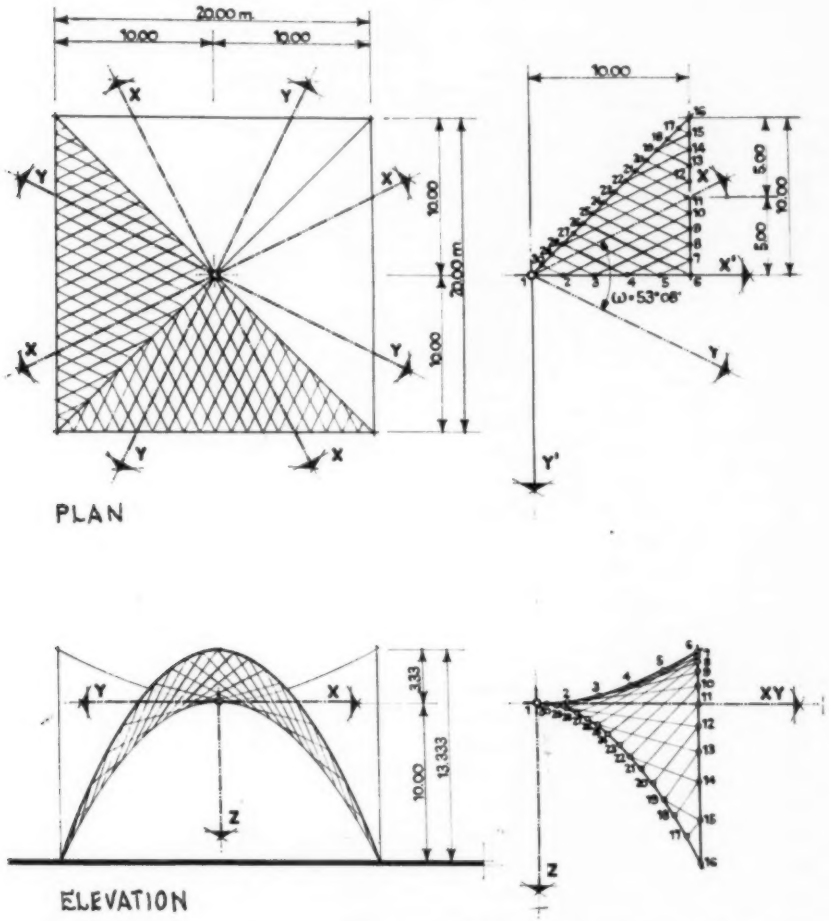


Fig. 8—Groined vault on square plan formed by the intersection of two hypars with vertical axes

3. The Edge 6-16 is left free of normal and tangential stresses, since there will be no structural element able to resist them. Through the use of Eq. (24a) and (24b) the values of $\bar{\sigma}_x$ and $\bar{\sigma}_y$, annulling normal and tangential stresses σ_β and τ_β in the Edge 6-16, are determined (Table 2, Columns 9 and 10). For this we need to know the values of α and β at the points 6-16, which are obtained by means of Eqs. (7) and (8) (Table 2, Columns 7 and 8).

4. Applying Eq. (4), the values of $\bar{v}_x$ and $\bar{v}_y$ are found for these points (Columns 11 and 12).

5. Find the values of the functions of integration $f_1(y) = \Delta v_x$ and $f_2(x) = \Delta v_y$ at the points 6-16 as differences between the basic values of v_x and v_y (Table 1,

TABLE 1

Point	1	2	3	4	5	6
	Y, m	X, m	Z, m	τ, kg per m	p_x, kg per m	p_y, kg per m
1	0.00000	0.00000	0.0000	750	450	450
2	1.11803	1.11803	—0.1333	757	450	450
3	2.23607	2.23607	—0.5333	776	449	449
4	3.35410	3.35410	—1.2000	808	449	449
5	4.47214	4.47214	—2.1333	850	449	449
6	5.59017	5.59017	—3.3333	901	450	450
7	4.47214	6.70820	—3.2000	923	420	533
8	3.35410	7.82624	—2.8000	986	446	660
9	2.23607	8.94427	—2.1333	1083	525	820
10	1.11803	10.06231	—1.2000	1205	648	1022
11	0.00000	11.18034	0.0000	1346	808	1199
12	—1.11803	12.29837	1.4667	1501	994	1406
13	—2.23607	13.41641	3.2000	1665	1200	1618
14	—3.35410	14.53444	5.2000	1836	1419	1835
15	—4.47214	15.65248	7.4667	2013	1646	2055
16	—5.59017	16.77051	10.0000	2194	1879	2276
17	—5.21749	15.65248	8.7111	2065	1748	2122
18	—4.84481	14.53444	7.5111	1938	1619	1970
19	—4.47214	13.41641	6.4000	1812	1492	1817
20	—4.09946	12.29837	5.3778	1688	1367	1665
21	—3.72678	11.18034	4.4444	1566	1245	1515
22	—3.35410	10.06231	3.6000	1447	1126	1366
23	—2.98142	8.94427	2.8444	1331	1012	1220
24	—2.60875	7.82624	2.1778	1220	902	1078
25	—2.23607	6.70820	1.6000	1115	799	942
26	—1.86339	5.59017	1.1111	1017	705	813
27	—1.49071	4.47214	0.7111	930	621	696
28	—1.11803	3.35410	0.4000	856	550	596
29	—0.74536	2.23607	0.1778	799	496	518
30	—0.37268	1.11803	0.0444	762	462	467

Columns 5, 6) and the ones found in Step 4 (Table 2, Columns 11, 12). These are represented in Columns 13 and 14 of Table 2.

6. When basic stresses at the border points 6-16 are modified, those of the Groin 16-30-1 are also modified, since Δv_x and Δv_y must be constant throughout the length of the generators. Values Δv_x and Δv_y of the functions $f_1(y)$ and $f_2(x)$ appear in Columns 15 and 16, Table 3, and the modified values of $\bar{v}_x$ and $\bar{v}_y$ in Columns 17 and 18.

7. With Eq. (4), $\bar{v}_x$ and $\bar{v}_y$ are transformed into $\bar{\sigma}_x$ and $\bar{\sigma}_y$ respectively (Columns 19 and 20).

TABLE 2

Point	7	8	9	10	11	12	13	14
	α	β_1	$\bar{\sigma}_x$, kg per m	$\bar{\sigma}_y$, kg per m	$\bar{v}_x$, kg per m	$\bar{v}_y$, kg per m	Δv_x, kg per m	Δv_y, kg per m
6	45° 11′	112° 35′	— 901	— 901	— 901	— 901	—1351	—1351
7	46° 17′	106° 12′	— 832	—1024	— 923	— 923	—1343	—1456
8	49° 29′	102° 18′	— 804	—1209	— 936	— 986	—1432	—1646
9	54° 23′	101° 13′	— 805	—1456	—1082	—1092	—1607	—1963
10	60° 29′	102° 33′	— 827	—1756	—1205	—1205	—1854	—2207
11	67° 19′	105° 34′	— 865	—2094	—1346	—1346	—2154	—2544
12	74° 31′	109° 38′	— 917	—2458	—1502	—1502	—2496	—2967
13	81° 43′	114° 12′	— 981	—2827	—1666	—1665	—2866	—3283
14	88° 40′	118° 56′	—1057	—3188	—1936	—1835	—3255	—3670
15	84° 46′	123° 33′	—1513	—2679	—2658	—1525	—4304	—3580
16	78° 43′	127° 54′	—2104	—2288	—3703	—1300	—5582	—3576

TABLE 3

Point	15	16	17	18	19	20
	Δv_x, kg per m	Δv_y, kg per m	$\bar{v}_x$, kg per m	$\bar{v}_y$, kg per m	$\bar{\sigma}_x$, kg per m	$\bar{\sigma}_y$, kg per m
16	—5582	—3576	—3703	—1300	—2104	—2288
17	—5156	—3580	—3408	—1458	—2003	—2479
18	—4730	—3690	—3111	—1700	—1898	—2786
19	—4304	—3283	—2812	—1466	—1784	—2310
20	—3955	—2907	—2588	—1242	—1712	—1877
21	—3605	—2544	—2369	—1029	—1632	—1488
22	—3235	—2207	—2129	— 841	—1541	—1162
23	—3126	—1903	—2114	— 683	—1605	— 900
24	—2996	—1646	—2094	— 568	—1663	— 713
25	—2866	—1456	—2067	— 574	—1728	— 615
26	—2742	—1352	—2037	— 538	—1784	— 614
27	—2619	—1343	—1998	— 647	—1826	— 708
28	—2496	—1432	—1946	— 836	—1845	— 882
29	—2382	—1607	—1886	—1089	—1840	—1116
30	—2268	—1854	—1806	—1387	—1795	—1396
1	—2154	—2154	—1749	—1749	—1749	—1749

TABLE 4

Point	21	22	23	24	25	26	27
	α	β_{σ}	σ_B, kg per m	τ_B, kg per m	Real interval, m	$F_{\sigma B}$, kg	$F_{\tau B}$, kg
16	78° 43′	152° 44′	— 636	1374	0.7984	— 508	1097
17	81° 30′	152° 39′	— 856	1385	1 5615	—1337	2163
18	84° 26′	152° 41′	—1156	1462	1.4919	—1725	2181
19	87° 33′	152° 51′	— 777	1273	1.4241	—1107	1813
20	89° 10′	152° 59′	— 488	1052	1 3588	— 663	1429
21	86° 41′	152° 28′	— 268	854	1.2964	— 347	1107
22	82° 02′	154° 01′	— 141	783	1.2384	— 175	970
23	78° 12′	154° 45′	— 40	637	1.1816	— 47	753
24	74° 13′	155° 32′	+ 17	518	1.1304	+ 19	586
25	70° 09′	156° 32′	— 1	426	1.0840	— 1	462
26	66° 04′	157° 37′	— 108	335	1.0432	— 113	349
27	62° 09′	158° 45′	— 304	228	1.C086	— 307	230
28	59° 00′	159° 48′	— 572	98	1.9808	— 561	96
29	55° 43′	160° 44′	— 886	— 67	0.9605	— 851	— 64
30	53° 48′	161° 23′	—1224	— 227	0.9481	—1166	— 215
1	53° 08′	161° 34′	—1624	— 375	0.4720	— 766	— 177

TABLE 5

Point	28	29	30	31	32	33
	$F_{H\tau B}$, kg	$F_{V\tau B}$, kg	$F_{H\sigma B}$, kg	$F_{V\sigma B}$, kg	F_H, kg	F_V, kg
16	633	896	— 334	236	299	1132
17	1306	1724	— 851	646	455	2370
18	1379	1690	—1057	862	322	2552
19	1200	1359	— 647	572	553	1931
20	991	1027	— 367	354	624	1381
21	805	759	— 179	190	626	949
22	740	627	— 83	98	657	725
23	601	454	— 20	27	581	481
24	489	323	+ 7	— 11	496	312
25	402	227	0	1	402	228
26	316	149	— 28	59	288	208
27	215	81	— 55	146	160	227
28	92	34	— 63	223	29	257
29	— 63	— 12	— 47	247	— 110	235
30	— 214	— 20	— 17	179	— 231	159
1	— 177	0	0	0	— 177	0

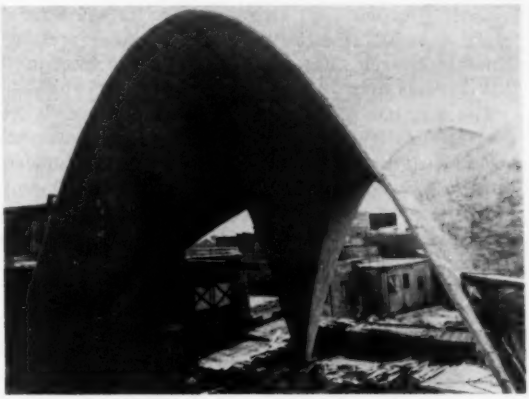


Fig. 9—Shell for San Antonio de las Huertas Church, Mexico City, is groined vault 50 x 50 ft. Thickness 1½ in. increased to 4 in. for a 2 ft width either side of groin. Architect: Enrique de la Mora

8. From Eq. (9a) and (9b), the values of σ_β and τ_β in the Edge 16-30-1 are obtained (Table 4, Columns 23 and 24). For this we must know before the values of α and β at these points (Columns 21 and 22) from Eq. (7) and (8).

9. Since σ_β and τ_β are stresses by unit length of edge, when they are multiplied by the real corresponding intervals (Column 25) forces $F\sigma_\beta$ and $F\tau_\beta$ concentrated in the points plotted at the edge are obtained (Columns 26 and 27).

10. Forces $F\tau_\beta$, contained in the plane of the groin, are separated into two components, one horizontal $F_H\tau_\beta$ and one vertical $F_V\tau_\beta$ (Table 5, Columns 28 and 29). $F_H\tau_\beta$ is taken as positive when directed from 1 to 16; $F_V\tau_\beta$ is positive when directed downwards.

11. Forces $F\sigma_\beta$ are acting along the plane normal to the Parabola 1-16 in each point, but being membrane forces must be contained in the plane tangent to the hypar at each point. That is, they are acting along the intersection of the plane tangent to the hypar with the plane normal to the Parabola 1-16 at each point. This means that, in general, their horizontal projections will not be normal to the projection of the Groin 1-16.

Once this direction is found we can solve $F\sigma_\beta$ in their vertical and horizontal components. The latter have in their turn one component normal and another tangential to the projection of the groin. Of these the normal is equilibrated by the forces coming from the contiguous symmetrical part of the structure. The remaining horizontal $F_H\sigma_\beta$ and vertical $F_V\sigma_\beta$ components appear in Columns 30 and 31 of Table 5.

12. Checking by vertical loads. Add up the vertical components of $F\tau_\beta$ and $F\sigma_\beta$ at the points of the Groin 1-16 (by which all the loads are transmitted to the support) and compare this with the total weight of the Triangle 1-6-16-1 as a first check that the calculations are correct. By the way, this is one of the ways to find the exact area of the surface, since it is not possible to directly integrate the double integral representing the area. Therefore, the check is only approximate and is performed by comparing the projected area with the real surface which is represented by the sum of vertical forces divided by g.

13. Calculate the Groin 16-1-16 with the loads transmitted to it from both sides. The forces acting on it will be those represented in Columns 32 and 33, Table 5, multiplied by two. The first column gives horizontal forces, the second vertical forces.

In this case the groin can be considered as a three-hinged arch, since its V section becomes almost flat at the crown, producing a practical hinge. It is

usually necessary to increase the thickness of the shell at both sides of the groin a small amount, to get a stronger member. In the groined vault of Las Huertas church (Fig. 9), the groins increase in thickness to 4 in., in a width of 2 ft on each side.

The number of columns in the tables has been abridged to give only the essential results of the calculations. The actual number of columns or steps in the calculation is of about 170 when performed manually with any ordinary desk calculating machine.

The use of electronic computers with adequate programming is especially appropriate for this kind of numerical analysis. This example was actually worked throughout with the IBM 650 machine of the Centro Electronico de Calculo of the University of Mexico.

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Title No. 57-17

Building for Economy with Hyperbolic Paraboloids

By GORDON MADSEN and DUTTON BIGGS

Designed for multiple use of moving forms, a hyperbolic paraboloid umbrella roof proved extremely economical for a shopping center in Minneapolis. Construction methods are described, as well as structural design features.

■ RE-USING FORMS ON A HYPERBOLIC paraboloid umbrella roof resulted in such a low cost that there can no longer be any doubt that this type of construction is by far the most economical of all framing systems in the range of spans used on this job.

The Bloomington Development Shopping Center, was designed using hyperbolic paraboloid umbrella shells for the roof (Fig. 1); the total roof area was 100,000 sq ft. Our bid was figured on a construction cost of $0.97 per sq ft, not including overhead and profit. This cost was for the column from the top of footing and the concrete shell in place.

Other projects built for this owner usually were designed in structural steel with a bay size of 20 x 34 ft. This concrete roof cost less, used less than half as many columns, inherently has great strength, will result in lower maintenance cost, is firesafe, and is giving the owner a substantial saving on his insurance. The following is an account of the construction methods, from the contractor's viewpoint, and some comments on the design by the structural engineer.

Details of the roof are as follows:

Building dimensions: 511 x 196 ft
Shell roof: four units wide and 11 units long for a total of 44 hyperbolic paraboloid umbrella shells 46 ft 4 in. x 48 ft 6 in. Shell rise—7 ft 9 in.; thickness—3 in.
Ceiling height: 14 ft
Column: 2-ft diameter

PROGRAM FOR ECONOMY

To get low construction costs we set up the following criteria:

1. *Schedule* — To achieve low cost the job must be scheduled so that all operations become an assembly line operation.
2. *Forms* — (a) Utilize a moving and shoring method that will result in a low first cost of the form. (b) Use a moving device that will result in

Gorden Madsen (ACI member) is vice-president of George Madsen Construction Co., Inc., Minneapolis. He has had 12 years' field experience in reinforced concrete construction. ACI member **Dutton Biggs** is a consulting engineer in Kansas City, Mo. Prior to opening his own office he was regional structural engineer for the Portland Cement Association with whom he was associated nearly 7 years.

a rapid form movement with a minimum of man-hours, but will not involve too much cost within itself.

3. Reinforcing steel — Pre-tie as much reinforcing steel as possible so that placing may be accomplished in minimum time.

4. Concrete — (a) Placing must proceed at the rate of two umbrella units per day. (b) Form stripping must be as rapid as possible without costly heating of concrete to accelerate strength gain.

CONSTRUCTION

The construction schedule was arranged to allow building of eight units per week using four sets of forms. One of the critical items was the length of time required to bring the concrete up to strength. Since the engineer's specification allowed form stripping at a job-cured cylinder strength of 3200 psi, we were able to use high-early-strength cement and attain this strength in 44 hr. We used a concrete mix with 7 sacks of Type III cement per cu yd and 3½ percent air entrainment. A retarding agent was used in hot weather.

For example, the two shells that were cast on June 8 were stripped on June 10. The shell on form No. 3 on which concrete placing was completed by 1:00 p.m., therefore, had a 44-hr curing period. The complete schedule is shown in Table 1. By examining Table 1 it can be

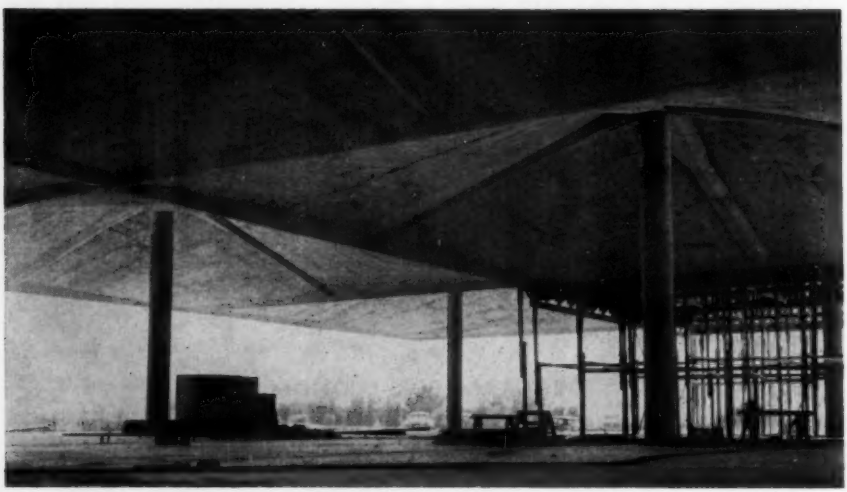

Fig. 1—Interior view shows the wide clear spaces and the clean lines of the hyperbolic paraboloid umbrella thin shell roof

seen that concrete was placed on eight shells per week and that the operation was back to the same starting place each Monday.

Fig. 2 — Forms were built on nailed trusses and beams were recessed

Forms

The key to economy lies in (a) economical use of material and (b) low forming cost.

The form was designed to span its own weight when supported on the moving devices at each end. However, when the concrete was placed on the form it was only necessary for the trusses to span the distance between each shore, which was 5 ft. Thus, a lightweight truss resulted that could be constructed entirely by nailing (Fig. 2). Since all generating lines of the double curved surface of the hyperbolic paraboloid are straight lines, the top chord of the trusses was straight and the 2 x 6-in. stringers were also straight.

The deck of the form was covered with ¾-in. plywood and the stringers were arranged so that as many full sheets of plywood as possible could be used (Fig. 3). Thus, when the form was disassembled there was a good stockpile of salvageable plywood with one unused face. The ¾-in. thick plywood warped easily to the curved surface and was held in place with screw-type nails.

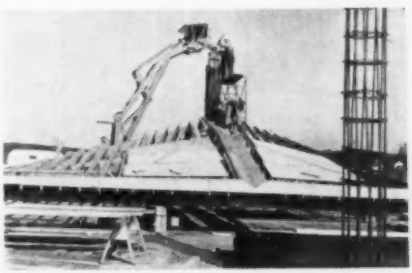

Fig. 3 — The 2 x 6-in. stringers were twisted so the plywood would fit. The stringers were placed to utilize full size plywood sheets and so were not always exactly parallel with the edge. (It should be noted that this does not alter the theoretical shape of the hyperbolic paraboloid as long as the stringer is tight against the top of the truss.) The carpenters found that it was easier to build the recessed beams first and deck over the shell areas last

The form was complicated by recessed beams, and took on the aspects of a cabinet maker's job. Since the carpenters were not familiar with building this type of form, a small crew was used at the start and they took their time figuring out all of the complicated beam intersections. Decking was mitered and joints were made to fit tight or were finally caulked so that a tight form would result. The first form required 2 weeks construction time with three carpenters. The fourth form was

TABLE 1 — SHELL CONSTRUCTION SCHEDULE

Day	Monday			Tuesday			Wednesday			Thursday			Friday			Saturday		
Time	8:00	12:00	4:30	8:00	12:00	4:30	8:00	12:00	4:30	8:00	12:00	4:30	8:00	12:00	4:30	8:00	12:00	4:30
Form No. 1	STR.	P.S.	——	P.C.	CR.	——	——	——	——	STR.	P.S.	——	P.C.	CR.	——	——	——	——
Form No. 2	——	STR.	——	P.S.	P.C.	CR.	——	——	——	——	STR.	——	P.S.	P.C.	CR.	——	——	——
Form No. 3	P.C.	CR.	——	——	——	——	STR.	P.S.	——	P.C.	CR.	——	——	——	——	STR.	P.S.	——
Form No. 4	P.S.	P.C.	CR.	——	——	——	——	STR.	——	P.S.	P.C.	CR.	——	——	——	——	STR.	——

Nomenclature: STR. — Strip and move form; time required—less than 4 hr.
P.S. — Place reinforcing steel; time required—approximately 3 hr.
P.C. — Place concrete; time required—4 hr plus.
CR. — Curing time was 44 hr. Membrane curing was used.

built in only 2 days construction time with seven carpenters. As the carpenters became familiar with the shape they were able to go much faster. The estimated cost of the form in our bid was $1.33 per sq ft and the actual cost was slightly less. Since each form was used 11 times, this came to 12.1 cents per sq ft of building area for the first cost of the form.

The form moving device (Fig. 4) was 13 ft 6 in. high and could be moved freely under the 14 ft clear ceiling. A crew of five men could easily move the form. To remove centering, lower, move, raise, and level the form and replace centering required less than 4 hr. Thus, it was possible for one crew to move two sets of forms in 1 day. This resulted in a very low cost per square foot for form moving.

One of the problems that worried us before we had experience with it was stripping the form from the concrete. The top side of the form was sprayed with a form oil and the edge and ridge beams had a slope of more than 45 deg which aided stripping. In most cases, the form dropped free of the shell by its own weight; sometimes it was necessary to drive wedges underneath one corner to start the stripping action. We noticed that the forms were easier to strip the second and third times than they were the first time. The longest stripping time required on any of the four forms on its first use was 1 hr. This time was reduced to 5 min or less for succeeding uses.

Construction operations

As soon as the first form was completed, it was erected into position and steel and concrete placing proceeded. At the same time the carpenters started building the second form. Concrete was placed on the first form May 13. The following week two shells were completed. The week of May 25 three shells were completed and the week of June 1 six shells were completed. Starting the week of June 8 and for succeeding weeks eight shells each were built. Thus, after three weeks the construction proceeded at the rate of 18,000 sq ft per week using four forms. Fig. 5-13 show the sequence of operations.

Placing reinforcing steel

To meet the schedule it was necessary to place steel on one roof section in 4 hr and on two roof sections in 8 hr. This was accomplished with a seven man crew. The edge beams and ridge beams were pre-tied. The steel over the column was partially pre-tied as shown in Fig. 10. A tension splice was introduced 6 to 10 ft from the top, so that the ridge beam steel could be slipped into place after the steel over the column was completed. Although distributed steel in the shell could have been pre-tied, it was not, on this job.

Concrete placing

The concrete placing was only slightly different from a conventional job. After the first shell or two was placed, the men established a system of placing in strips as shown in Fig. 13. The biggest secret in placing a shell of this kind is to keep ahead. The concrete was placed with a 2½-in. slump, vibrated, and leveled. Smoothing the top immediately with a float was all that was required, so it was easy to keep ahead once a system was set up. (If floating is delayed it becomes much more difficult and the results are not as good.)

Fig. 4—Four form lifting devices were specially fabricated for this job at a cost of $3400. The small wheels were a little hard to use but worked satisfactorily

Fig. 5—A general view (8:22 a.m.) showing the four forms. At 8:20 a.m. the testing laboratory reported a job cured cylinder tested 3270 psi so form stripping was started on form No. 1

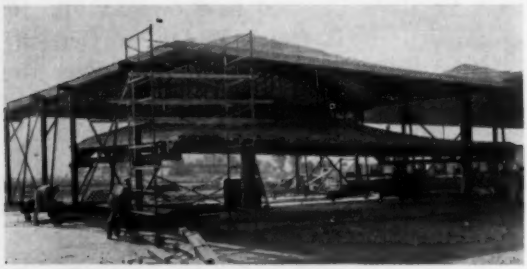

Fig. 6—Form No. 1, 8:50 a.m. — The form stripped easily from the shell

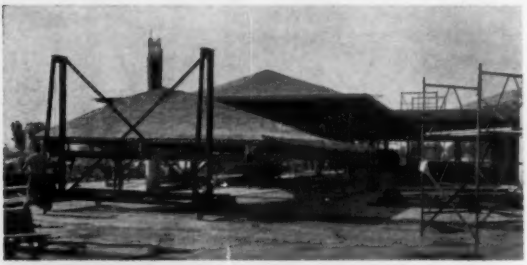

Fig. 7—Form No. 1, 8:57 a.m.—The first half is rolled forward. Immediately afterward the second half was lowered and moved forward

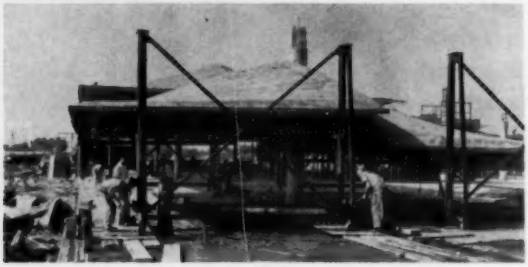

Fig. 8—Form No. 1, 9:17 a.m. — The second half is being raised. (The floor section under near quadrant was not in place and planking was used.)

Fig. 9—Form No. 1, 10:33 a.m.—Edges have been leveled, interior shoring is in place, and carpenter is building flat section at top around column

Fig. 10—Form No. 1, 11:42 a.m.—Steel placing begins. Steel for edge beams and ridge beams were pre-tied. Steel over column was partially pre-tied

Fig. 11—Forms No. 1 and No. 2, 4:00 p.m.—Shoring is in place. Form No. 1 is ready for concrete. Carpenter is finishing detail cut-out at top of form No. 2 and form will be oiled at same time. At 8:05 a.m. next morning steel placing began on Form No. 4

Fig. 12—Form No. 3, 8:06 a.m. — Concrete placing started

Fig. 13—Form No. 3, 11:52 a.m. — Concrete placing took more than 4 hr using two finishers and five laborers on top. In the background Form No. 4 is ready for concrete. By 6.00 p.m. concrete on Form No. 4 was in place

Curing

It is important that curing be started within 5 to 10 min after the final smoothing. If this is not done, plastic shrinkage cracks will almost surely appear. On this job, membrane curing was sprayed on.

Roofing

The roofing was bonded and was a three-ply built-up roof over 1 in. of rigid insulation. The felts contained asbestos fibers and a steep pitch asphalt was used. Gravel was used only in the valleys formed by joining the shells.

Drainage

Since the edges of the shells are level and are the lowest part of the surface, drainage was in two directions along the intersection of the edges.

DESIGN FEATURES

Design of the hyperbolic paraboloid roof followed the classic formulas as developed by Felix Candela.* Concrete design was based on 4000 psi and ultimate strength design methods were used. The design live load was 40 psf.

Deflections

One of the common problems with hyperbolic paraboloid umbrellas is excessive deflection of the corners. By turning the edge beams down as was done on this design (Fig. 14), the centroid of the edge section is below the center of the shearing action between the edge beam and the shell. This results in an eccentric bending condition in the edge beam that tends to lift the corners and thus to eliminate this deflection. This condition is somewhat similar to prestressing a beam. If the prestress force is below the centroid of the section (as it usually is), the center of the beam will lift off of the soffit. If the force were applied above the centroid (as is the case here), the ends of the beam would lift. In this case the force is introduced into the edge beam by the shell, and not by prestressing.

Deflection measured at the corner was ¾ in. while deflection at the midpoint of the edge was the same. This would indicate that the bending introduced by the eccentric load condition offset the bending produced by the dead load of the edge beam and the natural tendency for the corner to droop; with one condition offsetting the other the bending is reduced to near zero and the edge beam was therefore purely in compression.

*Candela, Felix, "Structural Applications of Hyperbolic Paraboloidical Shells," ACI JOURNAL, V. 26, No. 5, Jan. 1955 (*Proceedings* V. 51), pp. 397-416.

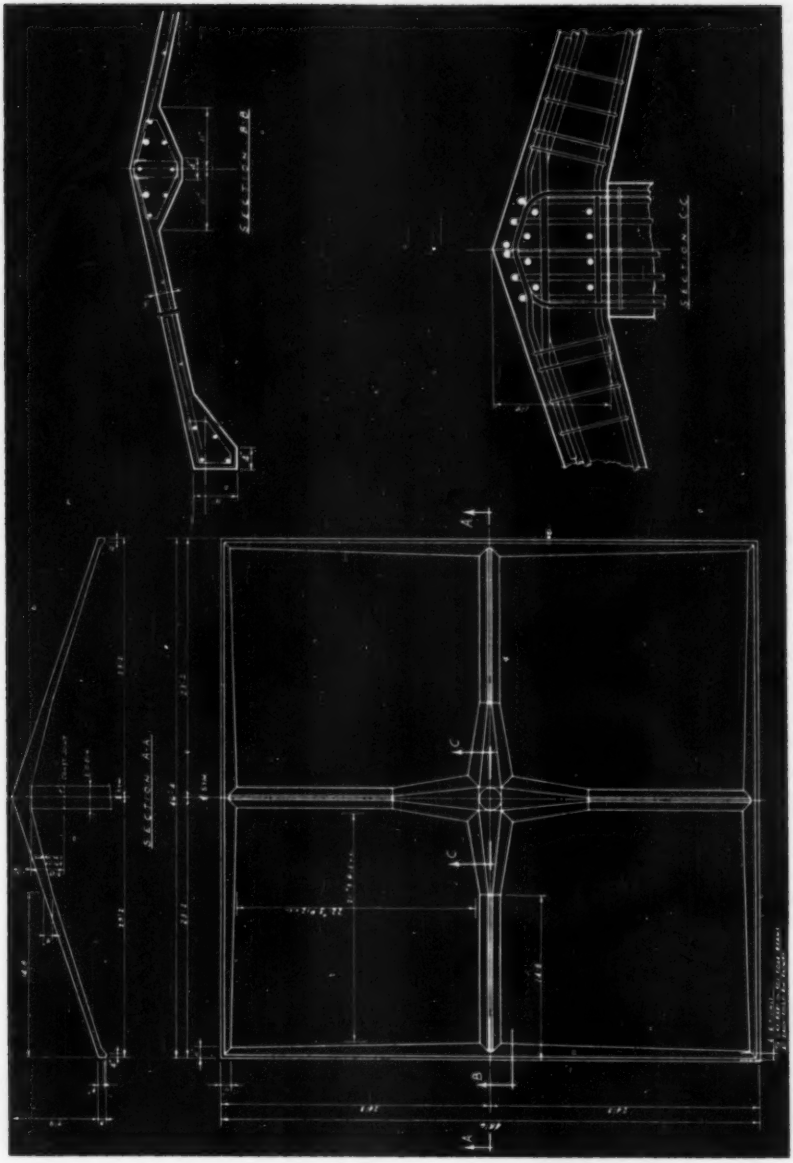

Fig. 14—Typical plan and details of roof framing, including sections through ridge beam and column

Unsymmetrical load

Unsymmetrical loads on the roof will cause bending in the column and bending and torsion in the beams adjacent to the column. Since there is no precise method of analysis for this condition in the shell and edge beams in the vicinity of the column, it was necessary to be somewhat conservative in the design. The coauthor observed a test in the structural laboratory of the Portland Cement Association on a 24 ft square hyperbolic paraboloid, which failed with an unsymmetrical load of 75 psf. The approach to the roof design was based on these observations.

A report on this test has recently been published in the Portland Cement Association publication *Reinforced Concrete* No. 35. It states: "This unsymmetrical load was gradually increased to 75 psf. No distress was observed over the major portion of the shell even as the ultimate capacity was approached. Cracking occurred at the interior edge beams and in the shell in the vicinity of the column. Cracks in the unloaded side of the structure occurred at the underside of the interior edge beam almost adjacent to the column. In the loaded side the cracks started at the top of the interior edge beams. Torsional cracks appeared in the other two interior edge beams. The 75-psf load applied over half of the structure produced a bending moment of 130 ft-kip, considerably more than the capacity of the two 15 x 15-in. interior beams. It is, therefore, evident that the participation of the 1½-in. shell acting as a deep V-shaped beam, and to a minor extent the torsion in the other two beams, were instrumental in carrying the unsymmetrical load. Hence, the recommendation that the unbalanced moment is resisted by two edge beams is extremely conservative."

It was felt that the best method of transferring the moment from the column into the edge beams was by the use of the loop bars shown in Fig. 14 and 15. These bars complicated the steel placing. To facilitate rapid placement of the steel and to eliminate threading of large reinforcing bars through the loop bars, a construction joint 3 ft below the shell was introduced in the column. This allowed placing of all ridge bar steel before the loop bars were placed. The roof itself averages

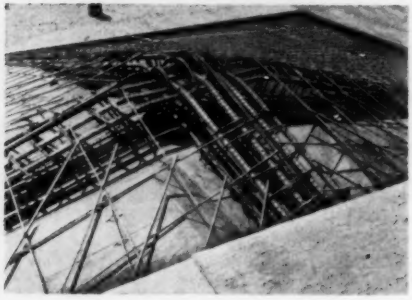

Fig. 15—Loop bars splice to column verticals and were placed after top and bottom ridge steel

2.25 lb of reinforcing steel (hard grade) per sq ft. The average concrete thickness of the roof is 4 in.

General arrangement

The building is 511 ft long and it was decided that two expansion joints would be introduced transversely. The groups of shells then, are 16, 12, and 16, for a total of 44. The edges of the shells are dowelled together with short #3 dowels. However, it is obvious that a tension build-up can occur down the valleys which are in a straight line 196 ft long. Dowels at the end of each edge beam make the reinforcement of these beams continuous in both directions.

CONCLUSION

Felix Candela was right when he said, "This is the most economical thing we have found." It is not only true in Mexico but it is also true in the United States with high cost labor. This type of construction — low in cost and high in quality — should meet with great acceptance in the U. S.

CREDITS

Owner of the shopping center is Bloomington Development Co. Architests were Manuel Morris & Robert E. Sixta, Kansas City, Mo. Structural engineer was Dutton Biggs, Kansas City, and supervising engineer was Ben Mayeron, Minneapolis. Contractor was George Madsen Construction Co., Inc., Minneapolis.

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Title No. 57-18

Hyperbolic Paraboloidal Umbrella Shells under Vertical Loads

By HOWARD P. HARRENSTIEN

Tests were performed on two reinforced concrete hyperbolic paraboloidal umbrella shells. The shells were subjected to a concentrated load at various selected points and resulting strains were observed. Stresses corresponding to these strains were determined, and the results are presented in the form of stress contours for each of nine selected load points. Superposition methods are presented which will predict the final stresses in an appropriate umbrella shell when it is subjected to any vertical loading provided the stresses remain within the proportional limit, and provided that deflections are small.

■ DURING RECENT YEARS ONE of the most widely used shapes for thin shell construction has been the hyperbolic paraboloid. These shells consist of one or more units which have the form of a warped surface that is generated by straight lines. Such shells have been used as roof structures, column footings, wall footings, floor systems, and various architectural motifs.[1,2] Many publications have been written on the design of the hyperbolic paraboloid, but few have been presented which treat the structural action of these shapes under loads other than uniformly distributed ones.

From the investigation reported herein, experimental data were developed which will enable designers to predict stresses in specific hyperbolic paraboloidal umbrella shells when these structures are subjected to vertical loads which are either uniform or nonuniform, concentrated or distributed, symmetrical or nonsymmetrical. The hyperbolic paraboloidal umbrella shell is one composed of four segments of the typical warped surface (Fig. 1). Shells built to this shape may either have stiffening beams along the valleys and exterior edges or they may be self-stiffening, i.e., without edge beams. This latter type of umbrella shell was the subject of this investigation.

EXPERIMENTAL INVESTIGATION

Two reinforced concrete hyperbolic paraboloidal umbrella shells were constructed as a class project by architectural engineering seniors at Iowa State University. The shells were 10 ft square in plan, 1 in. thick, and had a maximum rise of 1 ft 8 in. in 5 ft (Fig. 1). The ultimate

The work reported by **Howard P. Harrenstien** (ACI member), associate professor of civil engineering, University of Arizona, Tucson, was the result of research conducted at Iowa State University, Ames. Dr. Harrenstien was previously assistant professor of civil engineering at Iowa State and from 1958 to 1959 was on a National Science Foundation Science Faculty Fellowship. His chief interests are in the field of creative structures — specifically plate and shell structures.

compressive strength of the concrete at 28 days as determined from tests on 3 x 6-in. cylinders varied from 6850 to 7930 psi, with an average of 7500 psi. The modulus of elasticity varied from 4.26×10^6 psi to 5.07×10^6 psi, with an average of 4.75×10^6 psi, while Poisson's ratio averaged 0.17. Reinforcement was a mesh of No. 9 wire placed 2 in. on center each way with the wire parallel to the straight-line generators. The modulus of elasticity of this steel varied from 29.5×10^6 psi to 33.6×10^6 psi with an average of 32×10^6 psi. An "effective" modulus of elasticity of the combined concrete and steel was computed as 4.98×10^6 psi by assuming that the steel and concrete underwent similar strains. This effective modulus was used in converting measured strains to stresses.

The two umbrella shells were bolted to an 8-in. steel column and were loaded simultaneously by a single loading mechanism (Fig. 2) which produced the effect of a single concentrated load on each shell. The details of this loading mechanism may be seen in Fig. 3. A 10-ton portable hydraulic jack was used to apply the thrust which was measured by an SR-4 load cell and balancing unit. The jack and load cell were mounted on a counterbalanced lever-framework which allowed free vertical and horizontal motion but held the jack in a vertical posi-

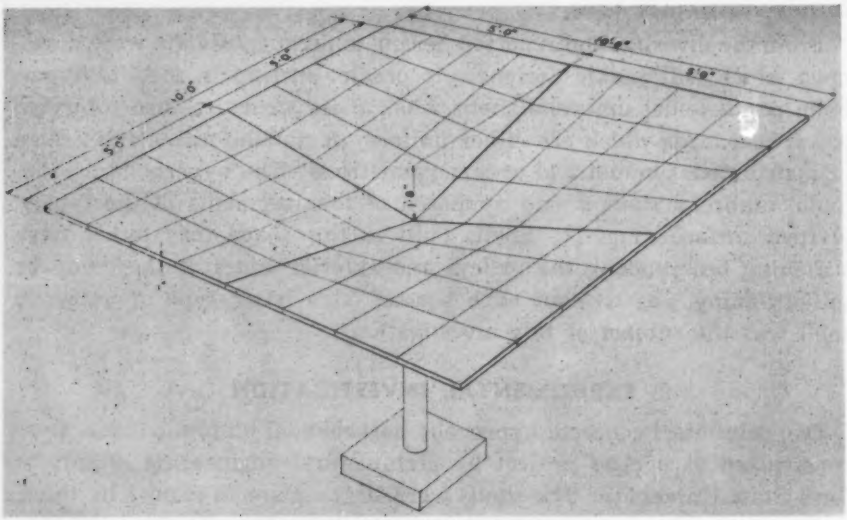

Fig. 1—Test shell dimensions

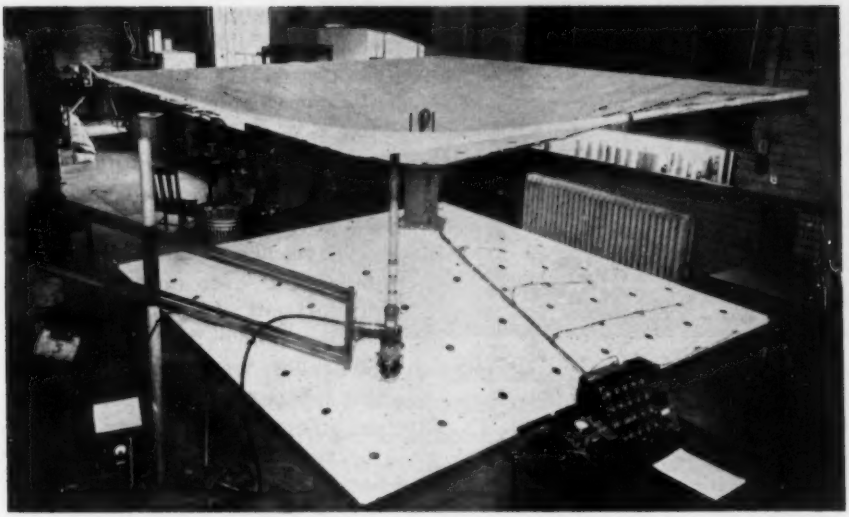

Fig. 2—Test set-up for shell loading

tion. No resultant vertical dead load thrust was exerted on either shell since the weight of the jack and load cell was balanced by loads on the opposite end of this lever mechanism. The distance between the two shells varied considerably with the position of the load. Because of this, pipes and pipe fittings were used to extend the jack piston so that it would be of the proper length for a specific load position. For the transfer of the load to the shell at the point of contact, ball and socket joints were used. Wood block cushions were placed between the steel end plates of the socket joints and the concrete shells to avoid severe stress concentration close to the load.

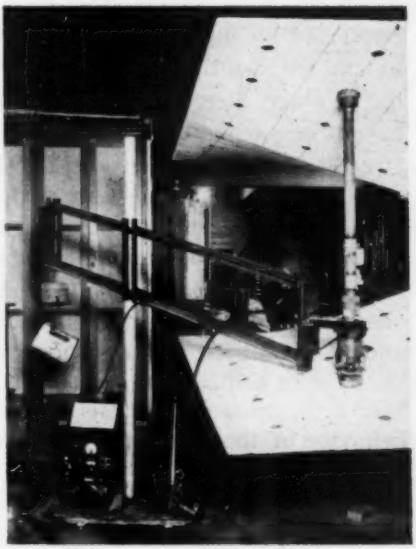

Fig. 3—Loading mechanism

At each of 60 selected points, the shells were loaded by the described mechanism in four equal increments to a maximum of approximately 548 lb. These 60 load positions correspond to the black circular spots in Fig. 4. Points D-4, D-5, E-4, and E-5 were not loaded because the loading mechanism would not fit between the shells at

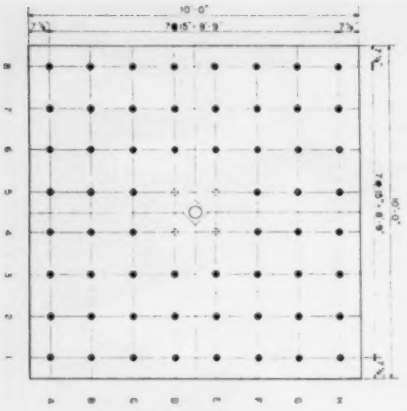

Fig. 4—Load positions

these points. Since these omitted points were close to the support, the stresses resulting from these loads were neglected.

For each of the 60 load positions, stresses were determined from strains which were observed at various key points by SR-4 resistance type gages. The gages were placed immediately over one another on both surfaces of each shell in the positions shown in Fig. 5. Linear type gages were placed on the edges where the directions of principal stresses were known, and rosette type were used at interior points. The placing of the gages immediately over one another made it possible to observe bending as well as direct strains. By so placing the gages over this ⅛ portion of the shell and observing their behavior as the increments of load were applied at each of the selected load points, strains at any point in the shell due to a load at some specific point could be predicted by geometrical superposition.

Separate load-strain curves were drawn for each gage for each of the 60 load points. Typical of these are the four shown in Fig. 6. The slopes of these curves were evaluated and the resulting values were converted by using appropriate relationships from mechanics to those having units of lb per in. per lb of load. Stress contours which indicate the average direct stresses are presented in Fig. 7 through 15. It is beyond the scope of this investigation to present the resulting bending stresses throughout the shell for the various load points. The complete details and data for the tests performed are in an unpublished Master's thesis[3] and may be obtained from the Iowa Engineering Experiment Station, Iowa State University, Ames.

It will be observed in Fig. 7 through 15 that values of σ_x and σ_y (resulting average normal stresses in x and y directions, respectively) and τ_{xy} (resultant average shearing stress on the xz or yz plane acting in the x or y direction) are listed as stress contours for a 1 lb vertical load acting through various points. The nine figures include the stress behavior of the shell under load at each of nine key points which are distributed over a ⅛ portion of the shell surface. By superposition principles, it is possible to use these stress contours to predict the final σ_x, σ_y, and τ_{xy} stresses at points away from the support due to any concentrated load or distributed vertical load (which necessarily must

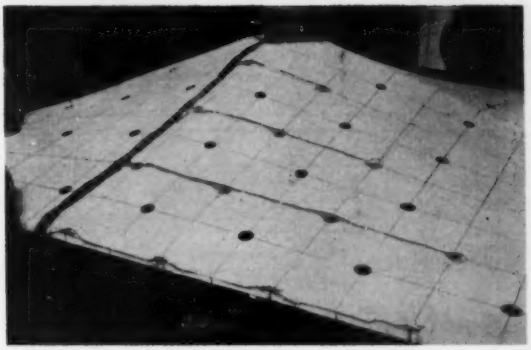

Fig. 5—Placement of linear and rosette SR-4 gages

be broken down into a series of concentrated loads). Principal stresses may be computed from these stresses by usual equation of mechanics.

To illustrate the superposition procedure, it is convenient to apply it to an example of an umbrella shell which is subjected to two concentrated loads. Assume that the final σ_x, σ_y, and τ_{xy} stresses are desired at point C-3 (Fig. 16) for an umbrella shell which is of the same dimensions as the test shell but which is loaded opposite to that of the shells tested. The concentrated loads to be applied are a 400 lb vertical load through point A-1 and a 200 lb load through point F-2. The complete picture of the shell is then one which opens upward and which supports the two concentrated loads that act downward through the designated points.

The magnitudes of the σ_x, σ_y, and τ_{xy} stresses at point C-3 due to the 400-lb load at A-1 would merely be 400 times the magnitude of the appropriate stresses which may be read directly from the stress contour chart in Fig. 7. The values of σ_x, σ_y, and τ_{xy} may be read from the figure as 0.095, 0.095, and 0.000 lb per in., respectively. Multiplying by 400, one obtained respective values of 38.0, 38.0, and 0.0 lb per in. Because the shell in this example is to be loaded opposite to that of

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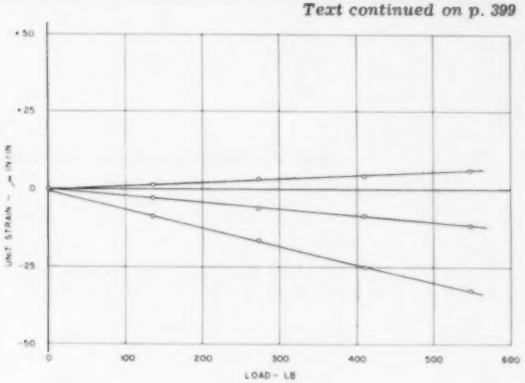

Fig. 6 — Typical load-strain curves

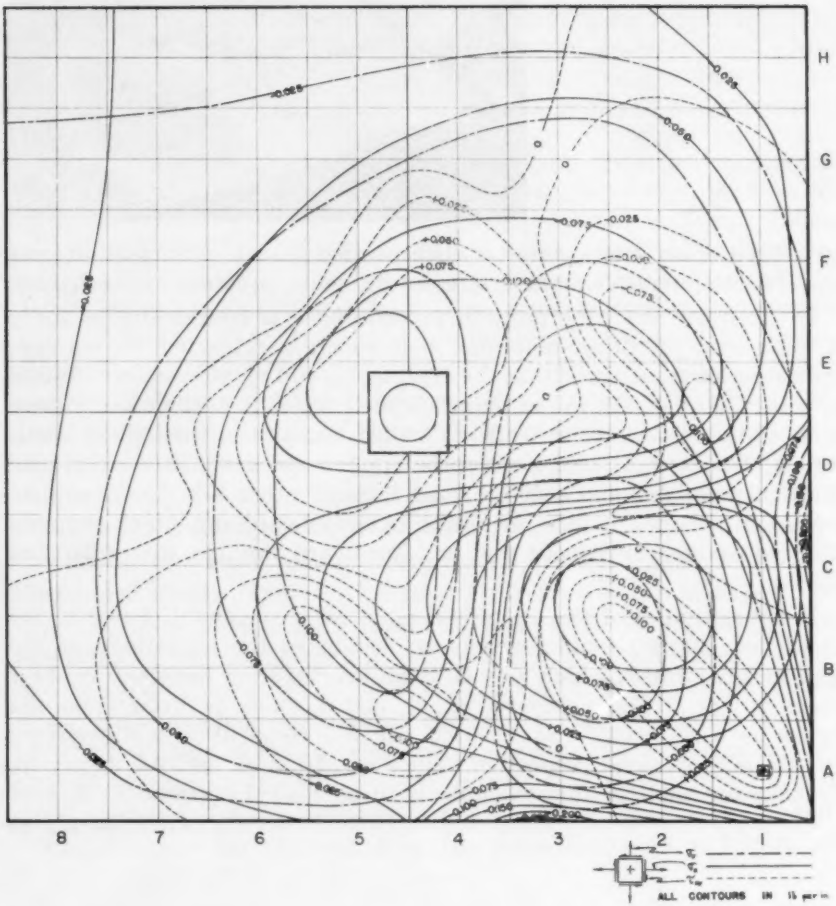

Fig. 7—Stress contours for 1-lb load at A-1

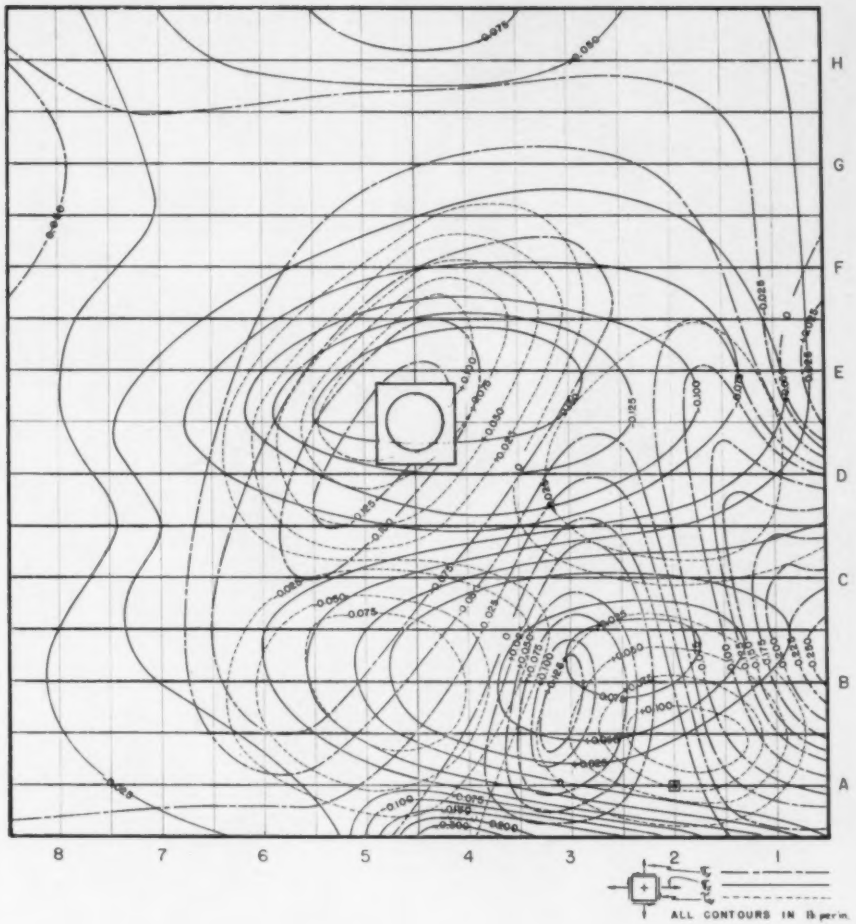

Fig. 8—Stress contours for 1-lb load at A-2

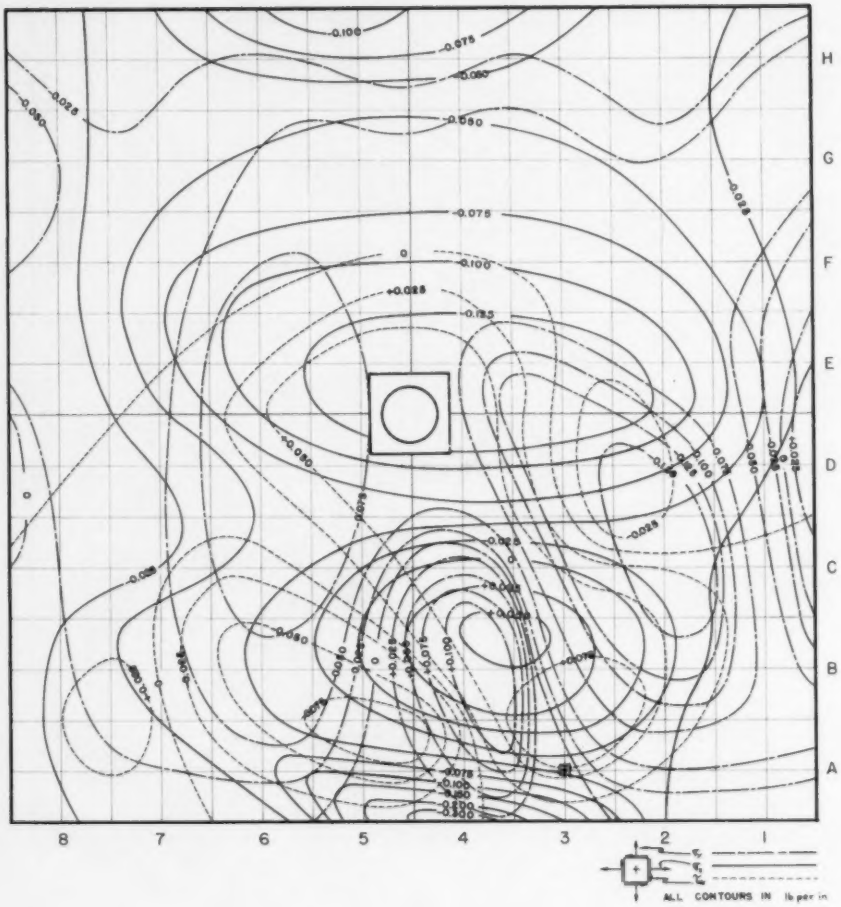

Fig. 9—Stress contours for 1-lb load at A-3

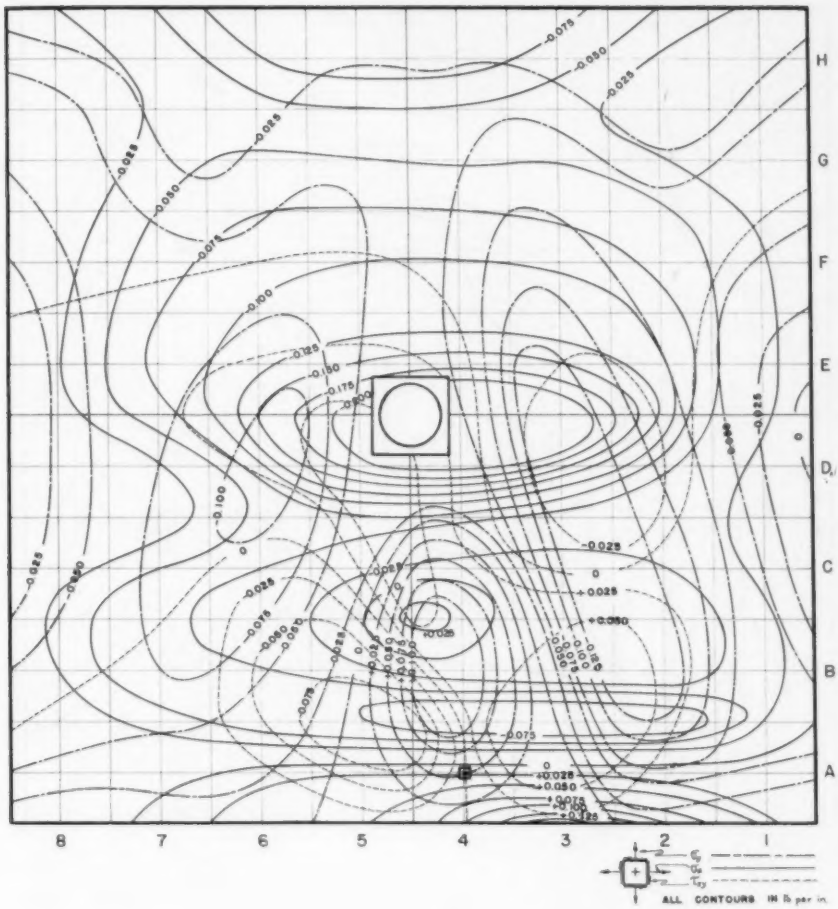

Fig. 10—Stress contours for 1-lb load at A-4

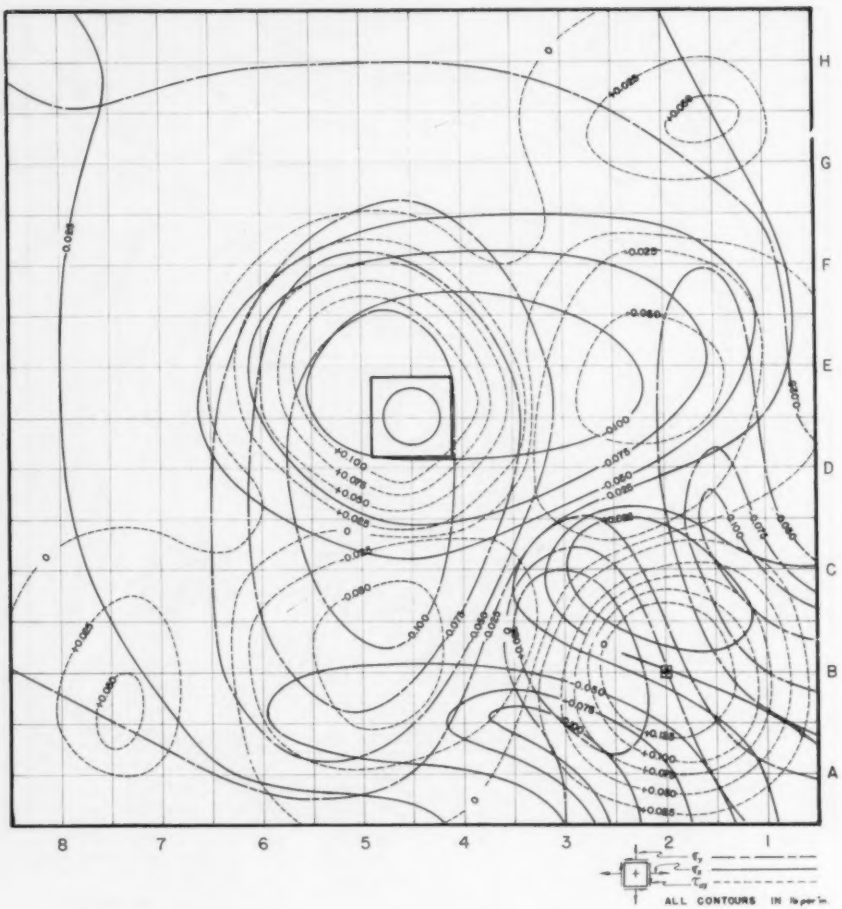

Fig. 11—Stress contours for 1-lb load at B-2

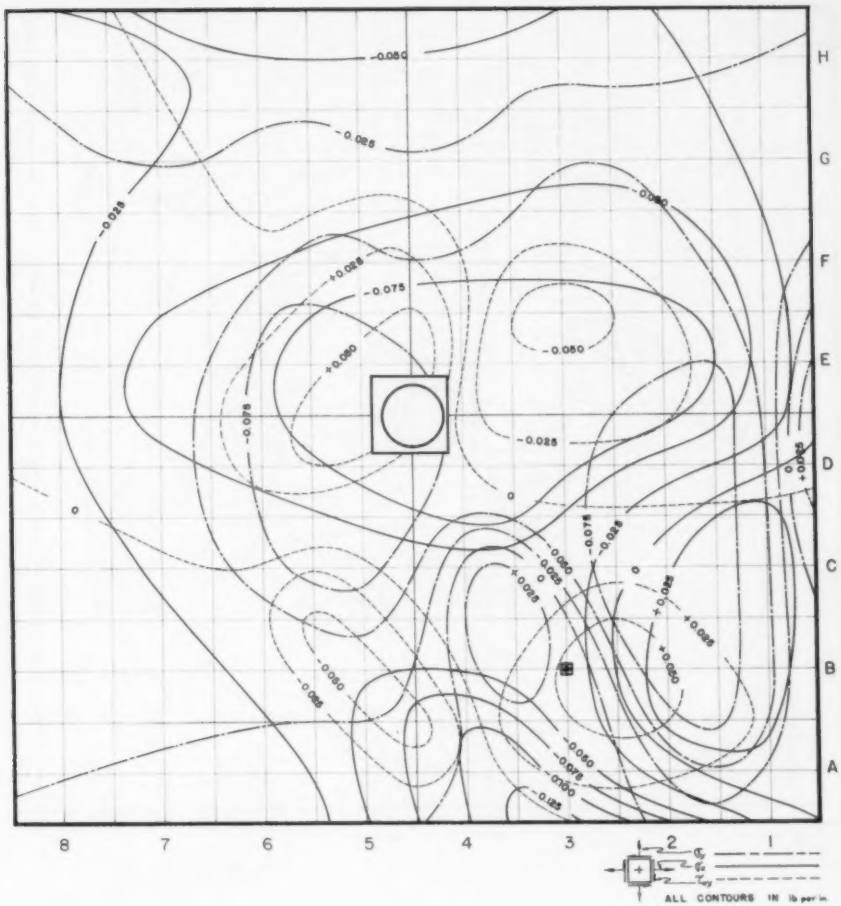

Fig. 12—Stress contours for 1-lb load at B-3

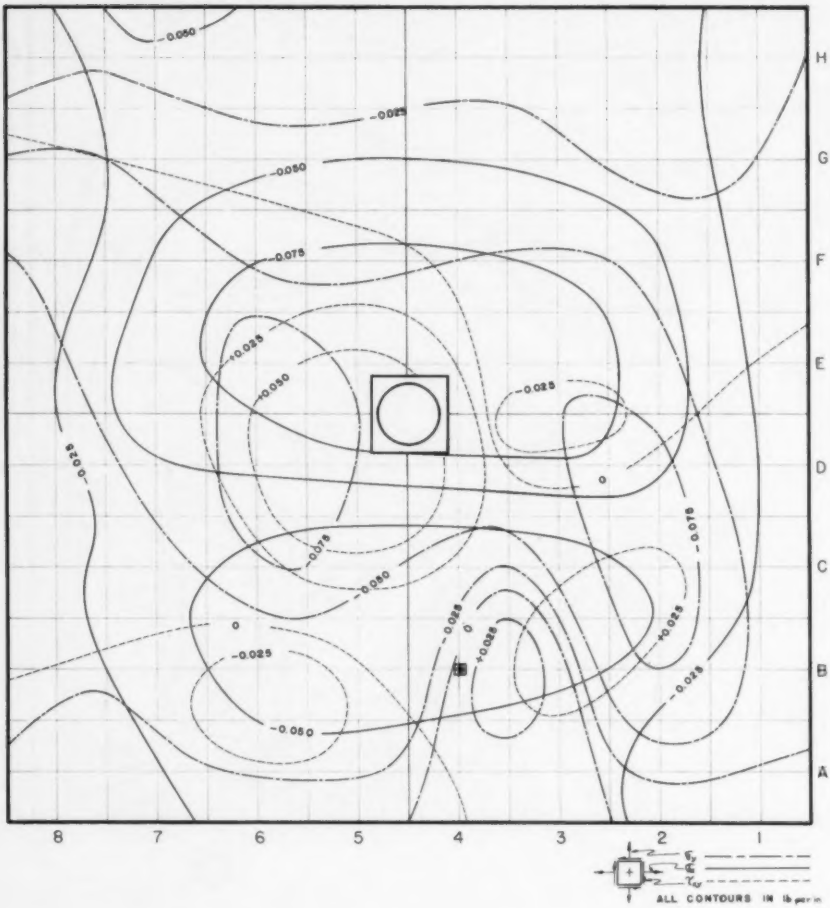

Fig. 13—Stress contours for 1-lb load at B-4

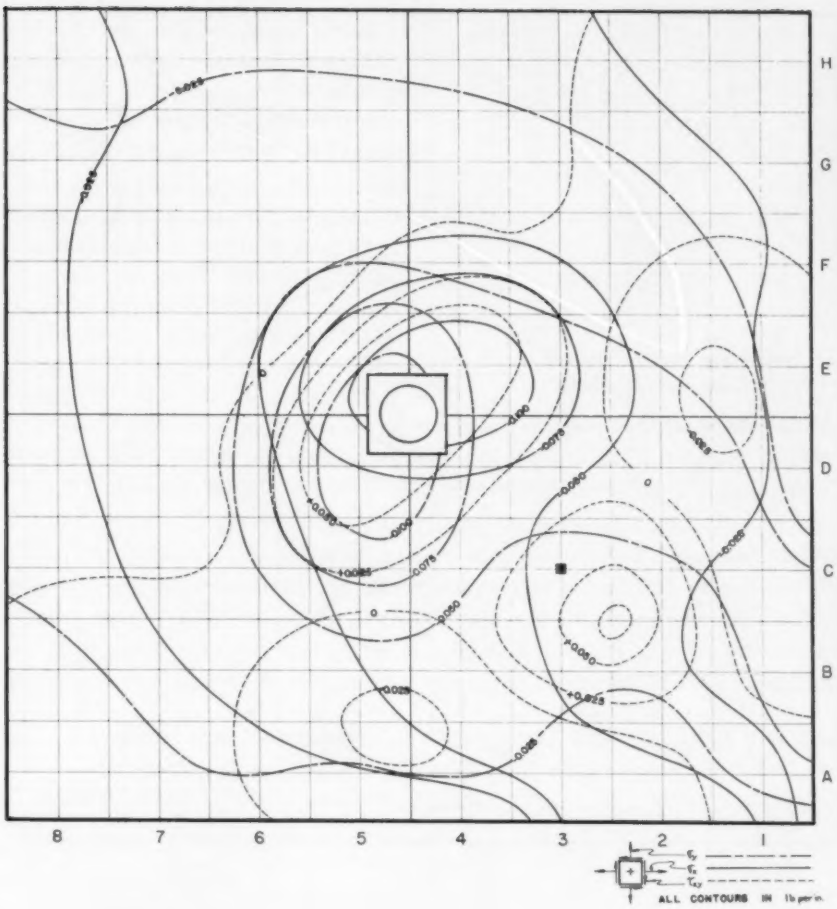

Fig. 14—Stress contours for 1-lb load at C-3

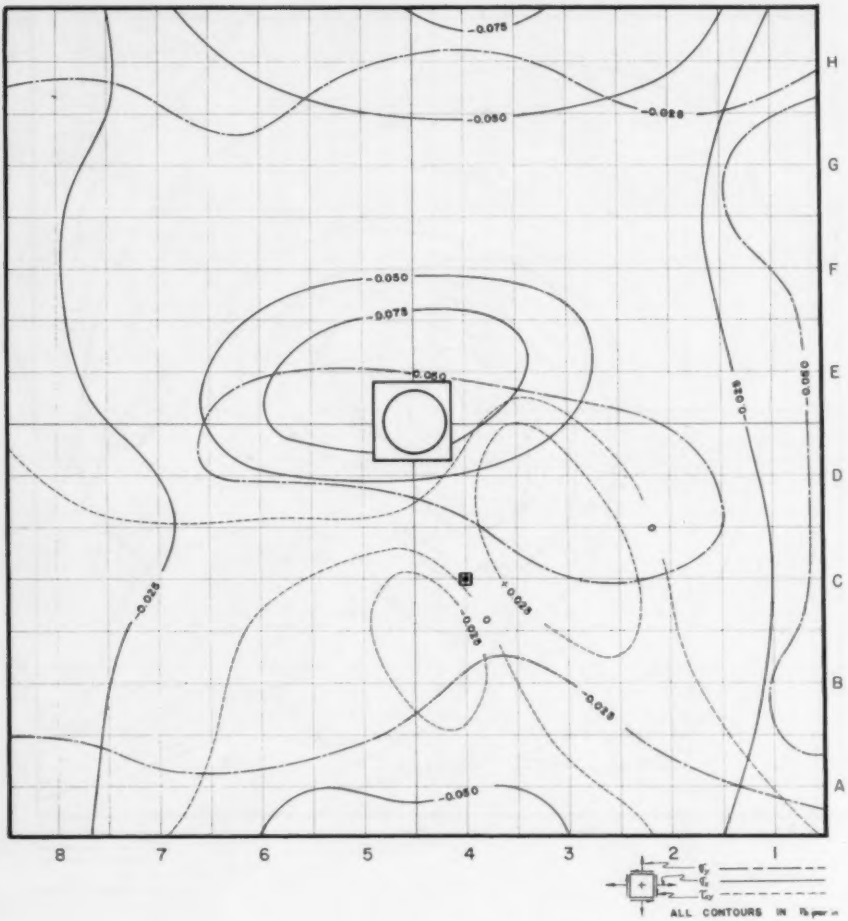

Fig. 15—Stress contours for 1-lb load at C-4

the test shell, it is necessary to change the signs of the stresses accordingly. The final σ_x, σ_y, and τ_{xy} stresses at point C-3 due to the action of the 400-lb force at A-1 would then be —38.0, —38.0, and 0.0 lb per in., respectively, acting according to the direction of positive quantities as shown in Fig. 7.

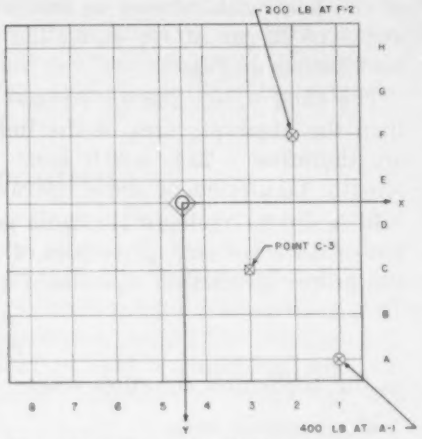

Fig. 16 — Example shell subjected to two concentrated loads

For the stress contribution of the 200-lb load at point F-2, geometrical superposition or rotation of axes must be employed. The magnitude of stress at point C-3 due to a load at F-2 is the same as that of the stress at C-6 due to the same load acting through B-3. It is necessary to observe, however, that a σ_x stress at C-3 due to the load at F-2 becomes a σ_y stress at C-6 when the load is at B-3 because of the rotation of axes which is involved. Similarly, σ_y at C-3 equals σ_x at C-6 and τ_{xy} at C-3 equals $-\tau_{xy}$ at C-6.

The resulting values of σ_x, σ_y, and τ_{xy} at C-6 due to a 1-lb load at B-3 may be read directly from the stress contour chart of Fig. 12 as —0.040, —0.063, and —0.005 lb per in., respectively. Multiplying by 200 lb and correcting for rotation one obtains —12.6, —8.0, and 1.0 lb per in. as the respective stresses. Changing the signs because of the direction

TABLE I — SUPERPOSITION PROCEDURE FOR EXAMPLE

Point at which load is applied	A-1			F-2		
Magnitude of load, lb	400			200		
Point at which stress is desired	C-3			C-3		
Chart used to obtain stress	A-1 (Fig. 7)			B-3 (Fig. 12)		
Point at which stress must be determined on respective chart	C-3			C-6		
Stresses to be determined	σ_x	σ_y	τ_{xy}	σ_x	σ_y	τ_{xy}
Stresses under 1-lb load	+ 0.095	+ 0.095	0.000	— 0.040	— 0.063	—0.005
Stresses under given load	+38.0	+38.0	0.0	— 8.0	—12.6	—1.0
Correction for rotation of axes	—	—	—	—12.6	— 8.0	+1.0
Correction for direction of loading	—38.0	—38.0	0.0	+12.6	+ 8.0	—1.0

Final stress at desired point:

$$\sigma_x = -38.0 + 12.6 = -25.4 \text{ lb per in.}$$
$$\sigma_y = -38.0 + 8.0 = -30.0 \text{ lb per in.}$$
$$\tau_{xy} = 0.0 - 1.0 = -1.0 \text{ lb per in.}$$

of the load with respect to that of the test shell then yields 12.6, 8.0, and —1.0 lb per in. for σ_x, σ_y, and τ_{xy} for the contribution of the 200-lb load acting at F-3.

The final σ_x, σ_y, and τ_{xy} stresses due to the action of both loads are then the algebraic sum of the individual contributions; σ_x, σ_y, and τ_{xy} are therefore —25.4, —30.0, and —1.0 lb per in., respectively. A systematic tabulation of these operations may be found in Table 1.

In umbrella shells of the same general proportions as the shell tested but of different size, principles of similitude[4] may be applied to obtain the proper prediction equation for stress. This procedure yields

$$\sigma = \frac{P}{P_m} \frac{l_m}{l} \sigma_m$$

as the prediction equation where

σ = desired stress
σ_m = determined stress in the test model
P = a load on the shell to be designed
P_m = similar load on the test model
l = a length in the design shell
l_m = similar length in the model

For example, let us consider an umbrella shell whose loads are 20 times those of the previous example and whose dimensions are five times as large. The final stresses in this shell would then be computed as follows:

$$\sigma_x = (20)(\tfrac{1}{5})\ \sigma_{x_m} = 4(-25.4) = -181.6 \text{ lb per in.}$$
$$\sigma_y = (20)(\tfrac{1}{5})\ \sigma_{y_m} = 4(-30.0) = -120.0 \text{ lb per in.}$$
$$\tau_{xy} = (20)(\tfrac{1}{5})\ \tau_{xy_m} = -4.0 \text{ lb per in.}$$

all acting in the appropriate directions.

When distributed loads are present, it is necessary to divide them first into a number of statically equivalent concentrated loads which act through the points for which stress contours have been drawn. The method is tedious except for simple load distributions but fortunately involves only algebraic and geometrical manipulations to obtain predicted stresses for any vertical loading.

SUMMARY AND CONCLUSIONS

The investigation reported was entirely experimental and involved the measurement of extremely small strains, both tensile and compressive. The construction material was one which is inherently subject to minute shrinkage and tensile cracking. The values presented are of necessity affected by these factors in addition to the usual variations present in any experimental investigation. It is not known to what extent these items are present but quite likely the combined effect is small enough to allow reasonable predictions of stresses by the outlined procedure.

In conclusion, it is hoped that the material presented here will provide architects and engineers with further information on the behavior of umbrella shells. This information should facilitate design of these shells for loads other than uniform. As a result, increased confidence in the use of such structures should be realized.

ACKNOWLEDGMENTS

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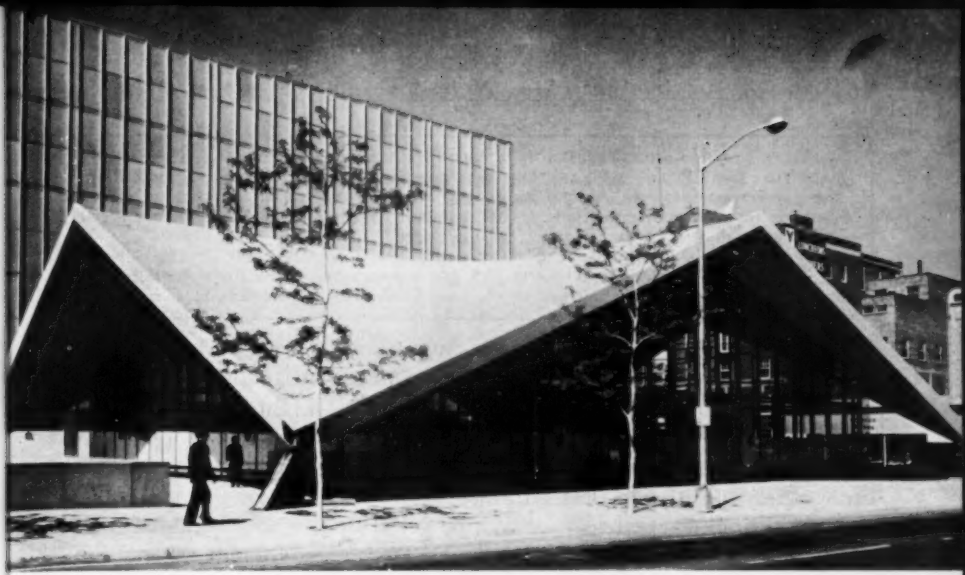

Title No. 57-19

Shell at Denver – Hyperbolic Paraboloidal Structure of Wide Span

By ANTON TEDESKO

A free spanning reinforced concrete shell gives an attractive shape to an exhibit pavilion for a Denver department store. The roof is made up of four hyperbolic paraboloidal surfaces and is supported through steel hinges on buttresses in the four corners of a rectangle, 112 x 132 ft; the 3-in. shell rises to a height of 28 ft. This paper describes design and construction features of the structure.

A HYPERBOLIC PARABOLOIDAL SHELL ROOF has recently been completed in Denver which increases the span of such shells beyond that of earlier applications. The structure was chosen by the architects, I. M. Pei & Associates, as a dramatic entrance feature for the new department store which is part of the Denver Court House Square Development. A modern hotel, an underground garage, and a flag-lined skating rink are other parts of this development by Webb & Knapp.

In current architectural usage "h.p." has come to mean "hyperbolic parabola," a phrase hard to pronounce. Thus, superior phonetical force

ACI member **Anton Tedesko** is a vice-president of Roberts and Schaefer Co., New York and Chicago, who are consulting engineers on major industrial, institutional, and government projects. Among the first to advocate the use of industrial production methods in shell roof construction, he has been directly responsible for the design and development of over 60 major shell projects.

Dr. Tedesko is chairman of ACI Committee 334, Concrete Shell Structures, and a member of ACI Committees 314 and 315. He is a member of the Reinforced Concrete Research Council and is chairman of its Task Committee on Composite Construction.

has made an "h.p. shell" out of a "hyperbolic paraboloidal shell." The h.p. shape and construction are less complicated than the name. Assume two parallel guide lines in space. If another straight line travels along these guide lines, the surface thus generated becomes a plane. Now assume that our straight lines are not parallel and do not intersect each other and that a straight line again travels along the two guide lines. A simple warped surface will be generated — the Germans call it a "ruled surface," as if produced by the simple movement of a ruler. If the travel of our straight generating line is uniform so that this line always remains parallel to a given plane, we end up with a saddle surface which is part of a hyperbolic paraboloid. Such a hyperbolic paraboloid could also be generated by a second system of straight lines, because there are always two sets of straight lines in such a doubly-curved h.p. surface.

Doubly-curved shells are much stiffer than those singly-curved. A singly-curved shell (a cylinder, a cone) requires stiffening members to keep it from "unrolling," so as to hold the shell to its original shape and thereby maintain its great carrying capacity. A doubly-curved shell (an egg, a dome) is a nondevelopable surface and cannot "unroll" because it is stiffened by its double curvature. Concrete can be cast in a doubly-curved mold as easily as in a cylindrical shape, but most of the great variety of doubly-curved shells are difficult to calculate and costly to form.

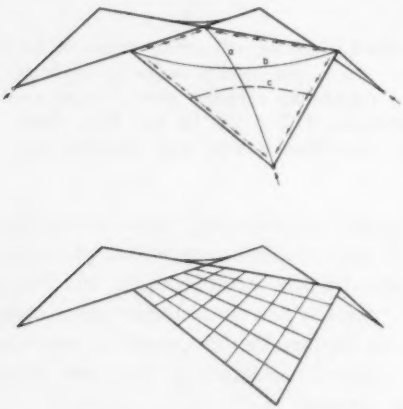

Fig. 1 — Diagrammatic view of hyperbolic paraboloidal shell

There is one notable exception: the h.p. shell. It is doubly-curved; it is inversely-curved; the curvature in one direction (convex) is of stiffening value for the other direction (in which the curvature is concave) and vice versa. The h.p. shell has good carrying capacity without extra stiffeners. It is comparatively easy to form, and the membrane stresses are not difficult to calculate. This means that for moderate spans, where

engineers are not much concerned about secondary stresses, it is a useful shape, as proven by the ingenious and inspiring structures designed by Baroni, Candela, and others.

DESIGN FEATURES

In this Denver shell, where the span is greater than in previous designs, secondary stresses had to be considered but the design and construction were still relatively simple.

The public enters the department store through the one-story, column-free exhibit building, which is covered by the shell structure made up of four h.p. surfaces. The roof is supported at four points in the corners of a rectangle, 112 x 132 ft, and rises to a height of 28 ft. The shape of the structure is shown in Fig. 1. The four h.p. surfaces are joined along ridge bands which form a big horizontal cross. Straight-line marginal ribs provide the vertical gable framing along each of the sides of the rectangle.

The shells are generated by sliding a straight line along the horizontal ridge as well as along the inclined marginal rib. Vertical planes parallel to the edges would therefore cut the shell along straight lines. Horizontal planes would cut the shells along hyperbolas (Line c). Vertical planes in diagonal direction would lead to convex parabolic arcs (Line a). A vertical plane through a wire stretched between peaks of adjoining gables would give a concave parabolic cut (Line b). This counter curvature and the direct compression and tension forces, for which these surfaces are known, help give the shell an extraordinary balance. (There is compression along Line a, tension along Line b).

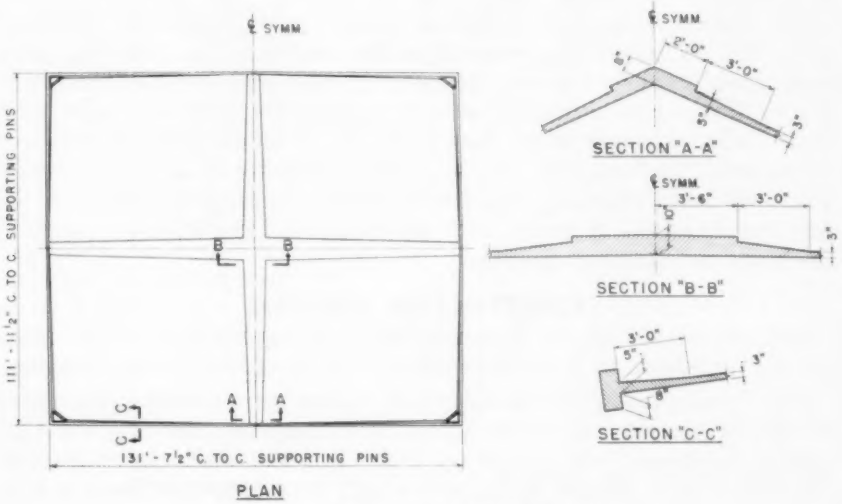

Fig. 2 — Dimensions of Denver shell

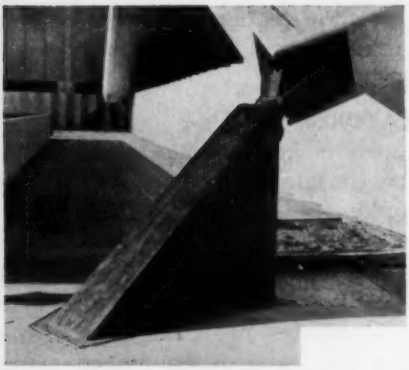

Fig. 3 — Corner detail showing concrete resting on steel shoe, hinge, and steel buttress

Shear forces are transferred from shell to ribs and ridges. The ribs and ridge bands, therefore, are subject to compression forces from the accumulating shear transferred from the shell. The compression in the gable members increases in downward direction. The ribs increase in section towards the corner, thereby keeping the unit stress fairly constant. The ribs are of rectangular section, 9 x 20 in. at their peaks, slightly twisted for architectural effect, remaining always at right angles to the tangential plane of the shell (Fig. 2).

The ridge bands take their greatest compression in the crown; here on account of lack of curvature a critical point exists from the standpoint of buckling. The architects did not want to provide projecting ribs along the ridges but wished to have a nonprojecting rib band. The rib band requires a good deal of reinforcing in the center where it is 10 in. deep, against 8 in. at the gables.

The shell has a minimum thickness of 3 in. and a basic reinforcing of two layers of #4 gage mesh with #3 diagonal bars in between. Its thickness is gradually increased to 5 in. toward the ribs over a distance of 3 ft. Reinforcing is added in this area of the shell, top and bottom, to provide for stresses due to flexure.

The shell is supported at the four corners by buttresses of structural steel (Fig. 3) which are connected to the steel framing of the two-story underground garage beneath, designed by engineers Weiskopf and Pickworth. The four corners of the shell are equipped with shoes of structural steel. Cylindrical pins of stainless steel provide hinges between shell and buttresses. The buttresses are oriented in the direction of the thrust, about 300 tons (including 40 psf snow load) acting approximately in the diagonal direction, at 35 deg with the horizontal. Fig. 4 shows details of the hinge and buttress assembly.

CONSTRUCTION FEATURES

The contractor built the falsework after getting his men familiar with the h.p. problems by first constructing a scale model of the centering.

Fig. 5 shows that 10-in. joints on 2-ft centers were placed along one of the sets of straight lines in the h.p. surface. Diagonal sheathing, 1 x 8 in., tongue-and-groove was nailed to the joists and took the curvature. The joists rested on 4 x 10-in. caps which were supported by 4 x 4 in. posts spaced at 10-ft centers, and screw jacks were recessed in the

bottom of these posts. Amount of camber was proportional to the anticipated deflection; a camber of 1½ in. was provided at the crown.

The reinforcing pattern for the shell can be seen in Fig. 6 to 8. The lines in the diagonal direction are the forming, the lines at right angles represent the steel which takes the principal tension.

The concreting operation was planned for July 3, to be completed prior to the July 4 holiday weekend. The temperature rose to 100 F on the preceding day, and the humidity was low. The engineers recommended that special precautions be taken for hot weather concreting, such as cooling of aggregates by inundation, addition of ice or a retarding agent, and of an admixture to reduce drying shrinkage. However, Webb & Knapp Construction Corp., the contractor, found it too late to change the mix. Instead, and as a concession to the heat, it was decided to start work at 4 a.m. when the temperature was 80 F.

Fig. 6 to 8 illustrate screeds, runways, and the concrete placing operation. A 6-bag mix with 3/4 in. maximum aggregate size produced the specified 3750 psi concrete strength which would permit decentering after 7 days. The concrete was placed simultaneously, starting from the four corners, by one crew in each quadrant. The concrete was furnished by 6-cu yd transit mixers. The two north quadrants could be reached by cranes (with 100-ft booms and 20-ft jibs) and concrete could be placed by bucket. Concrete for the south quadrants was hoisted in the adjoining multistory structure and carted over a runway to elephant trunks leading to the roof. For the upper portions of the shell, the buggies dumped directly onto the forms. Screeds were provided for the shell, but were not considered necessary because the layers of reinforcing, when properly placed and tied, furnished a sufficient guide to the concrete finishers.

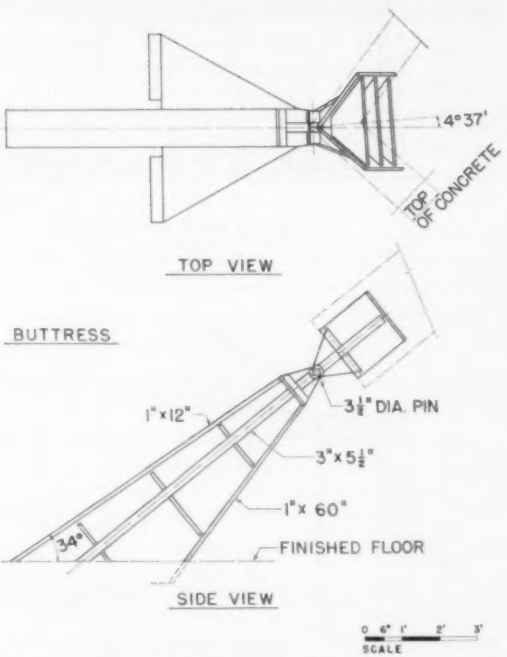

Fig. 4—Details of hinge and buttress assembly. Hinge is a cylindrical pin. Buttress is welded to steel beam of underground garage structure

The concrete lost water rapidly with increasing temperatures and the slump of the concrete at the truck had to be increased from 3½ in. at 4 a.m. to 5½ in. at 7 a.m. to 6½ in. at 9 a.m. Midday temperatures in the shade were 98F with no shade on the roof. The humidity was about 20 percent. Water hoses kept the concrete surfaces wet and prevented cold joints from forming. The concrete finish was good considering the weather conditions. The yardage placed was 225 cu yd, the placing time 10 hrs.

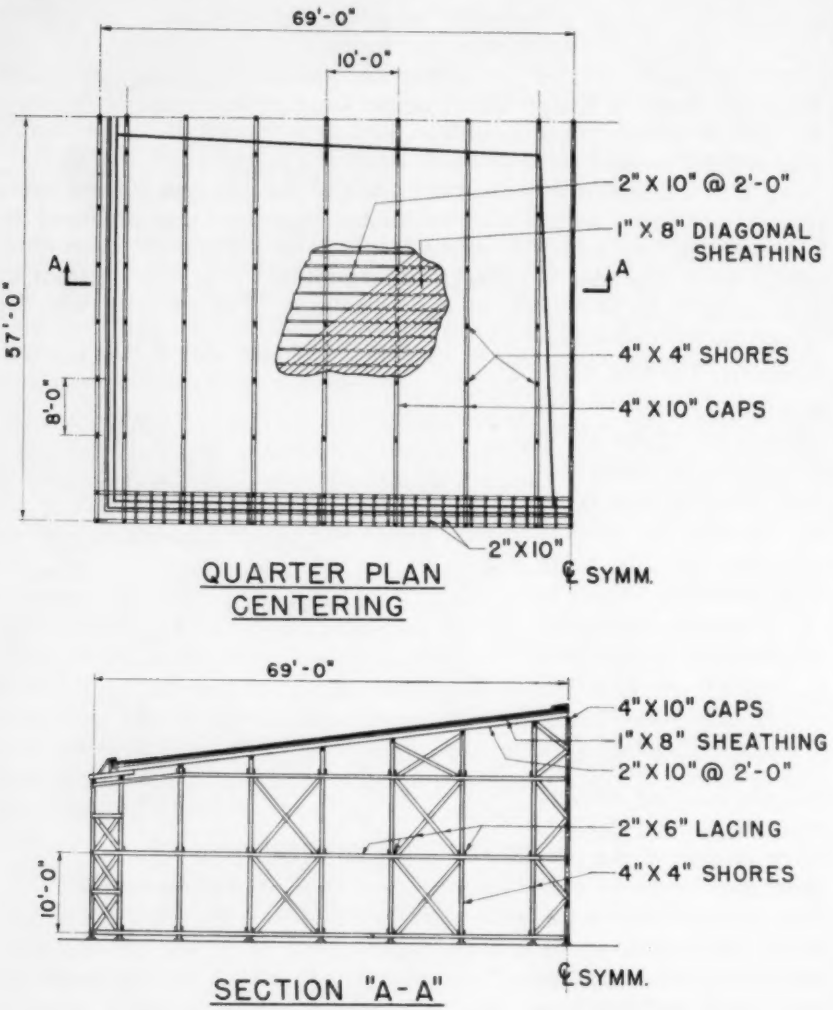

Fig. 5 — Details of centering. Contractor made model of this centering before building full scale structure

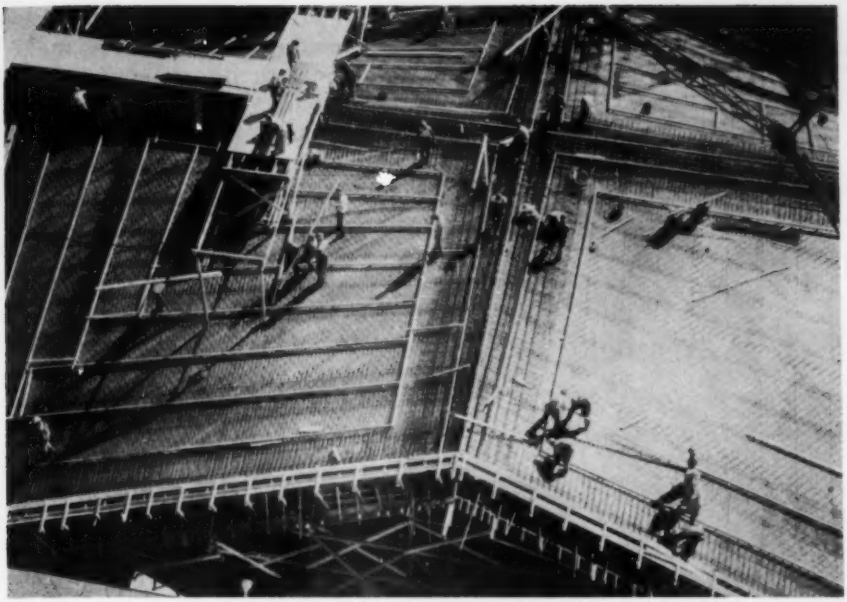

Fig. 6 — Top view of shell showing screeds and reinforcing. At left is runway for buggies; concrete at right was placed directly by crane bucket

The crews in each quadrant consisted of: four finishers, two vibrator men, two steel setters, two carpenters (working on runways, chutes, and screeds), and two men shoveling concrete, or a total of 12 men per quadrant. In addition there were on hand: two men at two cranes each, six men at buggies, four men at chutes, two men unloading buckets, one hoist operator, and one to four men on water hoses. A total of 35 laborers were used. Men handling buckets and vibrators switched to water hoses and to placing curing mats of fiber. Except for the heat, the job would have to be considered overstaffed; the workmanship was excellent.

On a job such as this when everyone is eager for a fast start with crews that have not yet gained experience, the first concrete is often placed too rapidly for sufficient vibration. This shell was no exception. The corners of the structure were flooded with concrete when it was still rather dark and inspection was difficult, resulting in some honeycombing in these areas.

The structure was decentered 14 days after the concrete placing. The 14-day strength was not known, the 7-day strength of the concrete was 2600 psi, and the modulus of elasticity 2,400,000 psi. The structure was decentered in the afternoon hours when the temperature was highest. Nine control points were located on top of the rib bands for the measurement of deflections from the adjoining store structure. Hinge movements were measured by micrometer.

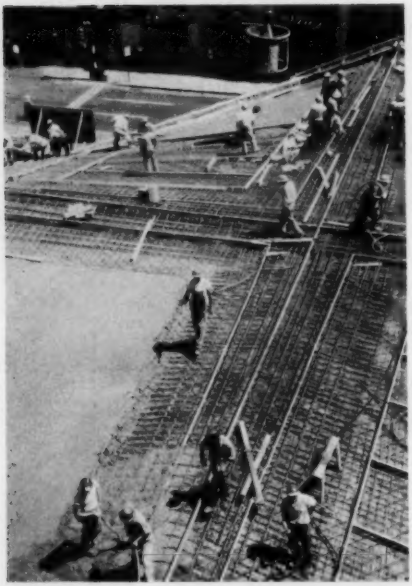

Fig. 7 — Concrete placing, also showing screeds and reinforcing of rib bands

Fig. 9 shows schematically the decentering sequence. The maximum measured deflection in the crown was 1½ in., close to the predicted value. Deflections at those points of the structure which are closer to ribs and hinges naturally were smaller. The deflection at the peak of gables was ½ in.

Shells take their worst punishment (have the least safety margin) at the time of the decentering when the concrete does not have its full strength, even though surcharge loads may be insignificant. It is important at such times to avoid stress concentrations, stress reversals, and resultant cracking, byproducts of an improperly planned stripping procedure.

A step-by-step plan of decentering, such as used here in Denver, assures stresses proportional or similar to those in the decentered structure. Carpenters lowering the screw jacks on which the scaffolding posts rested were divided into groups, and the sequence of operations was as follows:

> Jacks in Area 1 were completely loosened. Jacks in Area 2 received one turn, while those in Area 3 received one-half turn. The one-turn and one-half turn sequence was repeated as often as required to free the supports from the shell in Areas 2 and 3.

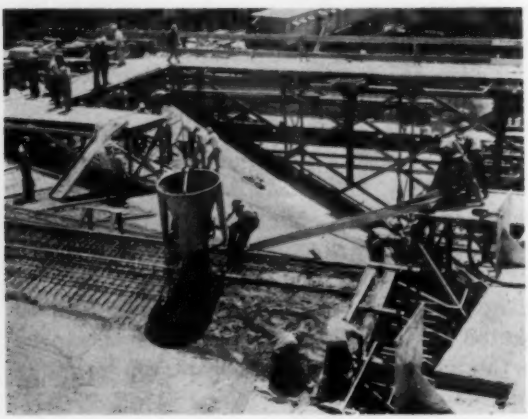

Fig. 8 — Concrete placing by bucket and buggy nears completion. Only rib bands in the crown remain to be cast. Runway in rear supports hoppers and elephant trunks which were used for placing concrete in lower portion of marginal ribs

Lowering operations (on one-turn and one-half turn jacks) proceeded in Areas 4 and 5 until the centering was completely free.

The turning direction for the jacks was painted on the floor to reduce the possibility of mistakes.

Marginal ribs of the gable, which for dead load and live load of the shell carry compression only, deflected 3/4 in. from a straight line between ridge and hinge, just like members in bending. They are bending members for the dead load of the rib plus the weight of the shell flange immediately adjacent. Also rib shortening might possibly have contributed to the deflection. Fine hair cracks developed at the top of the ribs at the peaks, such as might occur at the center support of a two-

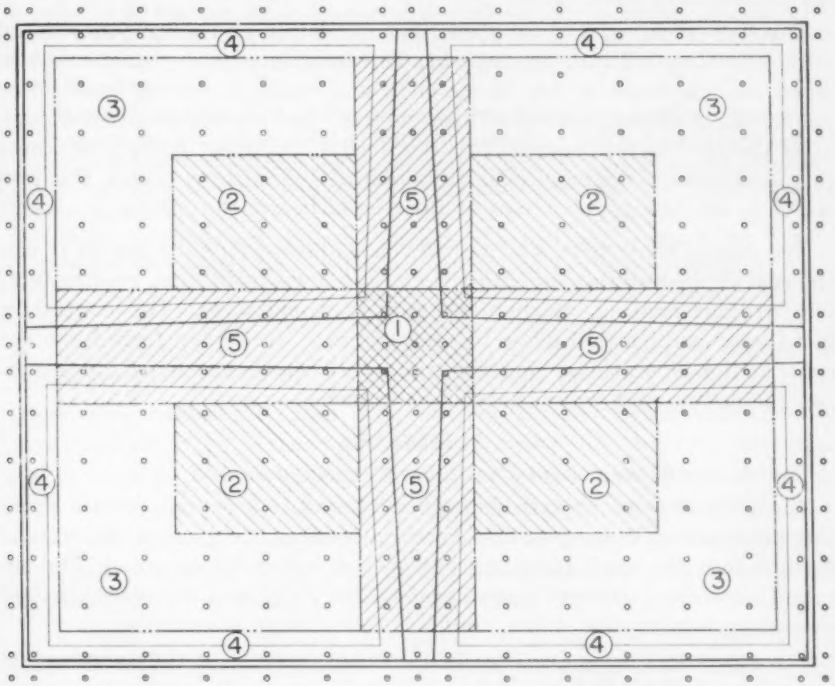

Fig. 9 — Plan of shell showing posts of form centering divided into areas for decentering sequence

Sequence of turning screw jacks at posts
Area 1 — 1 turn at a time, repeated until free
Area 2 — 1 turn at a time, repeated until free
Area 3 — ½ turn at a time, repeated until free
Area 4 — Center jacks 1 turn at a time, repeated until free
Corner jacks ½ turn at a time, repeated until free
Area 5 — Interior jacks 1 turn at a time, repeated until free
Exterior jacks ½ turn at a time, repeated until free

span beam. As a possible explanation, it could be said that the marginal ribs deflected downward because they shortened; however, the ridge bands forming the intersection of two shell quadrants prevented the peak of the gable frame from deflecting as much as it would have otherwise. Hair cracks resulted since the ribs were not able to move freely downward.

The interior of the structure received only a coat of paint. Vertical glass walls, located 17 ft from the shell edges, have slotted connections at roof level to provide for elastic and plastic movements. For each 10 deg F temperature difference, the calculated vertical movement of the roof was 1/8 in. It was therefore important for the sash contractor to consider the temperature of the day when the field measurements for his connections were taken.

The shell received 2 in. of glass fiber insulation and a built-up roofing with gravel rolled into the asphalt. The insulation was placed between wood nailing strips on top of the shell and made to run up against the rib bands with top surfaces flush so that the rib bands were hidden after the roofing was placed. This required an increase in shell thickness in a few places. Drainage was provided at the corners where the rain water spills into gravel boxes at floor level below the hinges.

The cost of the concrete structure was $3 per sq ft; $4.50 per sq ft was the cost of the building including floor, windows, insulation, roofing, etc., but without heating, mechanical, and electrical work. The contractor had the advantage that when in need he could draw men from his other construction operations. Had the roof shell been constructed by a subcontractor, the cost would have been higher.

SUMMARY

The structure was opened to the public August, 1958.

The shell stresses were calculated according to accepted theory. From test information it appears that bending stresses are greater and extend further into the shell from the edges than normally assumed. The so-called secondary stresses can therefore by no means be considered of secondary importance in h.p. shells of fairly large magnitude.

The structural design of the shell and supervision of the construction were furnished by Roberts and Schaefer Co., with the writer in charge.

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Title No. 57-20

Experiments with Thin-Shell Structural Models

By J. L. WALING and LONGIN B. GRESZCZUK

A new method of forming hyperbolic paraboloid concrete shell structures is presented. The method, which depends on the special geometric characteristics of the hyperbolic paraboloid, makes use of wire supported Styrofoam with its high strength to weight ratio as a temporary shell material to support the permanent shell material during curing. The Styrofoam thus serves temporarily as a structural material in lieu of extensive formwork and remains as insulation and vapor barrier throughout the life of the structure.

Experiments which led to the proposed construction procedure made use of space models, table sized construction models, and a laboratory construction model. Results of the model studies indicate possibilities of success with full scale constructions using the proposed materials and construction procedure.

Shells composed of skewed hyperbolic paraboloidal elements to form spherical and barrel type shapes are introduced for further study.

■ DURING THE PAST FEW YEARS structural engineers have shown an increasing interest in thin-shell construction of roofs covering large arena type buildings. This type of construction, which utilizes the structural strength of a continuous membrane for the roof or cover, makes possible a wide variety of new and pleasing architectural forms to cover spans unobstructed by girders, trusses, columns, or other structural supports. For the most part, the structures of this sort have been made of reinforced concrete in thicknesses varying from a low of about ½ in. up to thick shell proportions.

The high cost of forming thin shells of somewhat intricate shapes tends to make engineers turn away from these types of construction. To form a thin-shell roof of reinforced concrete has required a forest of falsework to support the curved forms for the concrete shell during its curing period. The formwork and the falsework have represented considerable cost which contributes nothing after the construction of the shell. Thus, the initial cost of forming has sometimes deterred engineers from designing shell structures.

Recent developments in the chemical industries are leading to the use of plastics for an increasing number of structural applications. The

aircraft and building industries are calling more and more on the relatively new materials for structural and semistructural uses.

For the past few years the structural engineering department of the School of Civil Engineering, Purdue University, has been engaged in research on the use of plastics in the construction of thin shell structures. Particular emphasis has been placed lately on studies of the use of Styrofoam* in forming concrete shells. The possibility of combining temporary structural functions with the excellent insulation and vapor barrier qualities of this material has been an exciting stimulus to progress on the project.

In accordance with the proposal for research submitted to the sponsor, the Dow Chemical Co., the project has consisted of three parts:

1. Experimental studies related to the fundamental mechanical properties of Styrofoam in its various densities and in sandwich type combinations with other materials such as concrete and reinforced plastics.
2. Theoretical research related to the effects on thin-shell theories of the mechanical properties of Styrofoam as the temporary structural material. These theoretical studies were directed toward the determination of practical construction procedures which could be made possible by the fairly high strength-density ratio of Styrofoam. Special attention has been given to shell shapes which seem to make especially advantageous use of the mechanical properties of the various densities of Styrofoam.
3. Experimental thin-shell model studies to determine the effectiveness of proposed construction procedures and to determine the strength, stiffness, and stability characteristics of the type of thin shells developed.

This paper will present primarily the results of the third part of the research program, i.e., results of the experiments with the thin-shell structural models.

SQUARE HYPERBOLIC PARABOLOID

Membrane stress analysis of hyperbolic paraboloid shells is well known. One has only to read of the works of Candela,[1] Aimond,[2] Pucher,[3] Reissner,[4] Strode and Dean,[5] and Parme,[6] to have a fair understanding of the membrane stress distribution and a notion as to its importance relative to bending stresses in the design of a hyperbolic paraboloid shell to withstand essentially uniform loading.

Geometric considerations

Much has been written on the geometry of the hyperbolic paraboloid surface, and much has been made of the fact that the surface can be generated by a series of straight lines. Indeed, the straight-line generators are the key to ease of forming a permanent hyperbolic paraboloid shell in the conventional manner of using falsework and wood forms. However, it is proposed that the true "magic" in the forming of a hyperbolic paraboloid structural shell lies not in its straight-line

*Trademark of the Dow Chemical Co.

ACI member **J. L. Waling**, professor (in charge), structural engineering, School of Civil Engineering, Purdue University, Lafayette, Ind., has been on the staff of the university since 1938 except for a brief tour as a naval architect at Norfolk Naval Shipyard, Norfolk, Va., in 1941-1945. He attained the rank of professor in 1954, and assumed his administrative duties in 1955.

Longin B. Greszczuk, associate engineer, Structures Division, Douglas Aircraft Co., Culver City, Calif., received his BSCE from Purdue University in 1958 and his MSCE in 1959. His master's thesis forms the basis for this paper.

generators but in the parabolas formed by the intersection of vertical planes and the hyperbolic paraboloid surface.

The hyperbolic paraboloid surface of Fig. 1, square in plan and oriented so that the coordinate axes X and Y are horizontal and Z is vertical, can be expressed by the equation

$$z = kxy$$

where k is a constant which is proportional to the surface's unitary warping, i.e., $k = h/ab$.

A rotation of the reference X and Y axes through a clockwise angle β about the Z-axis to the new position X_1, Y_1 as shown in Fig. 1(b) results in the following equation referred to the new axes:

$$z = k\left[\left(x_1^2 - y_1^2\right)\frac{\sin 2\beta}{2} + x_1 y_1 \cos 2\beta\right]$$

A vertical plane passed through the surface at $y_1 =$ a constant, say Q, intersects the surface in a curve whose equation is

$$z = \frac{kx_1^2}{2}\sin 2\beta + kQx_1 \cos 2\beta - \frac{kQ^2}{2}\sin 2\beta$$

Differentiation of this parabolic equation twice with respect to x_1 gives

$$\frac{d^2z}{dx_1^2} = k \sin 2\beta$$

Thus it is seen that for values of β between 0 and 90 deg the second derivative is positive and the curvature of the parabolic trace is concave downward, see Fig. 1(c). For values of β between 90 and 180 deg the trace is concave upward.

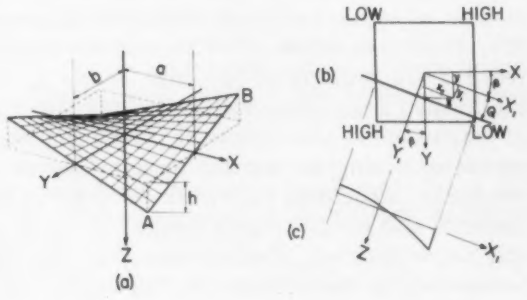

Fig. 1—Geometry of hyperbolic paraboloid

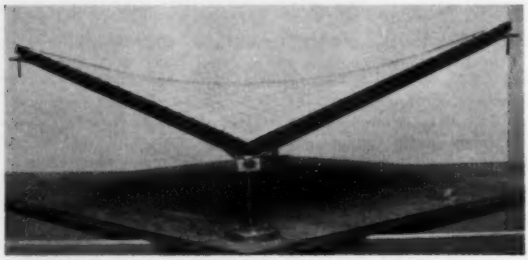

Fig. 2—View along plane of principal curvature

Since a parabola is the natural shape of wire subjected to a vertical load distributed uniformly in the horizontal direction, systems of wires stretched between edge beams in vertical planes parallel to the X_1Z-plane when β is between 90 and 180 deg can serve to hold up a thickness of Styrofoam. A set of wires placed at $\beta = 100$ deg and another set at $\beta = 170$ deg can provide a lattice of upward support for the layer of Styrofoam. Furthermore, a set of wires placed above the layer of Styrofoam at $\beta = 10$ deg along with a set placed at $\beta = 80$ deg provides a downward pressure on the layer of Styrofoam which can be used to assist in warping the straight planks of Styrofoam into the double curvature of the hyperbolic paraboloid surface. The amount of tension required in the wires to provide the necessary pressure (upward or downward) can be adjusted by changing the angle β to provide more or less curvature of the wires to fit the hyperbolic paraboloid surface. The more the curvature of the wires, the less is the wire tension required to provide a given pressure.

The warped layer of Styrofoam can be precompressed in its own surface by a system of wedges between the layer of Styrofoam and the edge beams. This precompression makes the warped Styrofoam layer act as a shell without the separate planks of Styrofoam being glued together. Such precompression of the Styrofoam tends to relieve the lower system of wires from additional stress under superimposed loads, by making the Styrofoam carry the main portion of additional superimposed loads by shell action. This allows the upper and lower lattices of wires to act as stabilizers (against elastic buckling) of the Styrofoam membrane. The interaction between the wires and wedges reduces the bending of the edge beams in the lateral direction. Thus, the formed Styrofoam shell can carry superimposed loads on the order of magnitude of the weight of permanent shell material, say reinforced concrete, during its curing period. The Styrofoam shell serves as a temporary structural formwork for placing and curing permanent shell material and would remain bonded to the permanent shell as insulation and vapor barrier. Furthermore, the Styrofoam can serve as a base for plastering the interior of the shell roof.

The third phase of this research program has dealt primarily with experiments on the type of construction made possible by the geometry of hyperbolic paraboloids as described above.

SMALL SCALE MODEL

Construction

In anticipation of experiments on a large scale laboratory model hyperbolic paraboloid shell, a small scale model was constructed. This small scale model was 37⅞ in. square in plan with a total rise of edge beams of 9.22 in. This table sized model was approximately scaled down from the proposed dimensions of a large scale model.

Each edge beam was made from two aluminum angle bars. The edge beams were supported at the high corners by ½ in. diameter aluminum rods and at the low corners by 1 in. diameter aluminum rods connected to a circular base. The latter columns supported the frame by a ball bearing arrangement which essentially provided a simple vertical support. In addition, a tie rod connected the low corners of the shell to resist the horizontal thrust which would normally be transmitted through the edge beams to abutments at the low corners.

S-33 Styrofoam, a fire retardant material having a density of approximately 2 lb per cu ft, was used to form the experimental shell. The basic units were planks 3¾ x ½ x 16 in. to be in proportion to the dimensions of Styrofoam planks to be used on the large scale laboratory model. HD-2 (high density) Styrofoam, which has a density of about 4.5 lb per cu ft was used for making the prestressing wedges.

Six-gage music wire was used to temporarily support and to stabilize and prestress the Styrofoam.

Two rows of equally spaced holes were drilled in each edge beam. Spacing between the rows was ½ in. while spacing along the edge beams was $1\frac{15}{16}$ in. The holes of two adjoining edge beams were fitted with machine screws each of which was drilled transversely near its end to receive a single strand of wire. Thus, the machine screws served to adjust the tension in the wire system. To resist any side movement of the screw, two nuts were placed on each screw, one on the inside of the edge beam and one on the outside.

The bottom wire lattice was formed by stretching wires between the two pairs of opposite edge beams. Each wire was skewed two spaces from the direction of the transverse straight-line generator so its parabolic shape (concave upward) when uniformly loaded with Styrofoam would conform with the shape of the hyperbolic paraboloid surface. One end of each wire was placed through the hole in its adjustment screw to provide individual control in the tensioning of each wire.

Before the Styrofoam planks were laid in place, a few top wires (skewed to be concave downward) were stretched to hold the planks in position. The planks were then laid loosely from the center outward toward the edge beams. These planks were laid with staggered ends in brick-like fashion to reduce the continuity of joints between the planks. The planks were cut to fit into the frame along the edges of the shell.

By offsetting the bottom wires in one direction and the top wires in the opposite direction, the bottom wires were exerting upward force while the top wires exerted downward force. This arrangement of wires forced the Styrofoam to take the shape of the hyperbolic paraboloid.

To develop strength and stiffness of the Styrofoam layer so that it would act as a structural shell, the planks were precompressed. Wedges were inserted

TABLE 1 — DEFLECTIONS OF SHELL DUE TO UNIFORM LOADS

Load		Dial gage										
Sand, in.	psf	1	2	3	4	5	6	7	8	9	10	11
¾	6.25	272	183	41	205	50	258	167	176	3	1	3
1¼	10.40	429	297	75	310	74	401	265	275	2	1	5
2	16.70	689	479	133	504	139	646	435	443	3	—1	6
2*	16.70	709	493	137	519	145	664	449	457	3	—1	6

Note: All deflections are given in thousandths of an inch. Downward deflections are considered positive.
*Deflections taken after load remained on the shell for 20 hr.

in pairs between the edge beams and the planks. Activation of the wedges and tensioning of the top and bottom wires precompressed the Styrofoam shell and practically eliminated bending in the edge beams. Precompression of the planks eliminated the use of adhesive. The individual planks were held by frictional force to resist shearing stresses and the precompression made possible the resistance of tensile forces. Fig. 2 shows the completed model.

Accuracy of construction

On completion of this model, measurements were made to determine the amount of deviation, if any, from the true hyperbolic paraboloid shape. The maximum deviation was found to be about 0.2 in. in the vertical direction.

Apparatus

In anticipation of measuring vertical reflections produced in the shell and one edge beam by various loads, dial gages were set up at a number of points under the shell. Fig. 3 shows the location of the dial gages. The gages were mounted on fixed stands, mounted in turn on three leveling screws, with the gage plungers engaging sheet metal covered wedges glued to the underside of the shell surface. Thus, vertical movements were indicated by the gages.

A wood frame resting on the edge beams was used to subject the shell to uniform sand loads. Two opposite sides of the frame were made from 6-in. boards and the other two sides were made from ¾-in. wood strips. This was the setup to obtain a ¾-in. uniform sand loading of the shell. Sand was placed on the shell and the excess was screeded off by moving a straightedge along the ¾-in. strips, always parallel to the 6-in. board.

Uniform sand loads of other magnitudes were obtained by fastening additional wood strips on top of the ¾-in. wood strip. However, for uniform sand loads in excess of ¾ in., it was found necessary to add another strip midway between the two ¾-in. edge strips. This was necessary since the greater sand loads caused the central portion of the shell to deflect downward, thus producing a thicker sand load at the center of the shell than at the edges. The uniformity of the load was checked by measuring the thickness of sand at various points on the surface.

Loading

Table 1 summarizes the deflections of the small shell due to complete uniform loads of ¾, 1¼, and 2 in. of sand. It can be seen that the center deflection as indicated by Gage No. 1 was 0.429 in. when the shell was subjected to 1¼ in. of sand. Thus, the maximum deflection was on the order of magnitude of the shell thickness when the sand load was about 2½ times the shell thickness.

To get some indication of the magnitude of deflection which might be caused by creep in the Styrofoam shell, the full uniform load of 2 in. of sand was allowed to remain on the shell for 20 hr. It can be seen from Table 1 that creep deflections during the 20 hr were practically negligible.

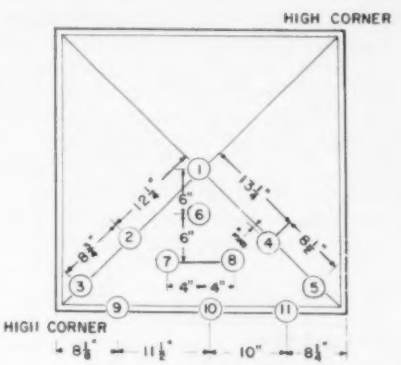

Fig. 3—Location of dial gages

Tests were undertaken to determine the loading sequence of the shell which would produce minimum deflections at the center of the shell during the course of loading the entire surface. Such an established sequence would be of value in determining the sequence of placing a permanent shell cover of concrete over the Styrofoam shell.

The shell was subdivided into 16 equal squares with ¾-in. high cardboard strips. These individual squares were given designations as shown by the numbers at the middle of each square in Fig. 4. Each square was then loaded in turn with ¾-in. of sand and the deflections

TABLE 2—DEFLECTIONS OF SHELL DUE TO PARTIAL UNIFORM LOADS

Area loaded	Dial gage										
	1	2	3	4	5	6	7	8	9	10	11
1b	—2	1	0	51	36	0	0	7	0	0	0
1a	—2	0	0	—5	0	—3	0	—2	0	0	1
2b	—4	—4	—1	—1	0	—6	—3	—4	0	—1	—1
2a	—5	40	41	—2	0	—5	10	0	3	3	3
3a	2	9	0	—5	—1	—3	0	—3	1	1	0
3d	—1	—7	—1	54	7	—6	—9	—5	0	0	—1
3c	5	6	—1	52	6	35	28	88	0	1	2
3b	—1	—3	—1	—5	—1	—3	—3	—2	—1	0	0
4a	—3	59	6	—2	1	1	11	0	1	1	0
4b	—1	61	5	—2	1	20	98	18	2	4	4
4c	—4	—14	—2	—5	0	—14	—12	—12	0	—1	—1
4d	—6	—13	—3	—2	—1	—11	—13	—8	—1	—2	—2
5a	71	—2	—2	—6	—2	12	—2	—2	0	0	0
5b	77	—14	—4	71	2	107	—4	52	—2	—3	0
6a	73	74	—1	—4	—1	93	59	9	0	2	2
6b	55	—29	—5	6	0	0	—22	—8	—2	—3	—2
Total	254	164	31	195	47	217	138	128	1	2	5

Note: All deflections are given in thousandths of an inch. Downward deflections are considered positive.

HIGH CORNER

4 1a	9 3b	5 4d	2 2b
12 3a	15 5a	13 6b	7 4c
8 4a	14 6a	16 5b	11 3d
1 2a	6 4b	10 3c	3 1b

HIGH CORNER

Fig. 4 (above)—Subdivision of shell and loading. Numbers in upper corner represent final loading sequence

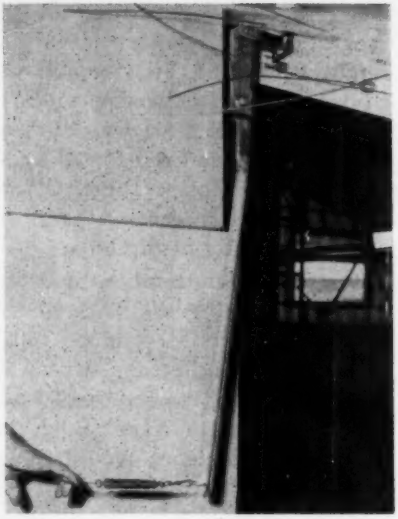

Fig. 5 (right)—Apparatus used in pretensioning of wires

indicated by the dial gages are shown on Table 2. Then a loading sequence was established which would produce minimum deflections at the center of the shell over most of the load application time. The shell was then loaded through that sequence with the resulting deflections as shown in Table 3.

Membrane stresses produced in the ½-in. thick Styrofoam shell by 2 in. of sand load would be approximately 8.3 psi; that is, the pure shear stresses in the directions of the straight-line generators of the surface as well as the tensile and compressive stresses at 45 deg from the generators would be about 8.3 psi. This is well within the useful range of stress in the Styrofoam.

LABORATORY MODEL

Construction

Space limitations in the laboratory dictated the maximum size of the square hyperbolic paraboloid construction model. This shell was 20 x 20 ft in plan with a rise of 7 ft from the low corners to the high corners. The low corners were placed on abutments extending about 4 ft above the laboratory floor, the two abutments being tied together by a tie rod to resist the outward thrust from the shell. The edge beams were chosen to be well overdesigned so that experiments could be performed on the Styrofoam shell or on a permanent shell cover without danger of failure of the edge beams. The edge beams were fabricated from rolled 6-in. steel Z-sections weighing 21.1 lb per ft. The high corners of the edge beam frame were supported by 4-in. steel pipe columns resting on the laboratory floor.

Holes ⅛ in. in diameter were drilled in the edge beams to take the top and bottom wires. The holes for the top wires were drilled parallel to the top flange of the Z-section and 1⅛ in. below it. These holes were spaced at 12 in. on centers horizontally. To correct for the slope of the shell surface in two

directions along the length of the edge beam, the exact distance of the bottom holes vertically below the top holes had to be calculated. The distance between the two sets of holes varied from 3.18 in. at the center of each beam to 3.38 in. at the ends. This arrangement of holes essentially provided the possibility that the top surface of the Styrofoam shell would be a true hyperbolic paraboloid. The bottom surface would have nearly the same shape and for all practical purposes, the difference was considered negligible.

Preliminary calculations indicated that a 3 in. thick surface of S-33 Styrofoam would be sufficient to withstand the stresses induced by the placement of a 2-in. cover of concrete. Thus, Styrofoam planks 24 in. x 3 in. x 8 ft were chosen as the basic size for the construction of the shell.

The size of the wires used in temporarily supporting and precompressing the Styrofoam was governed by two requirements: (1) they were to be of sufficient size so that standard prestressing equipment could be used to pretension the wires, and (2) they were to be of sufficient strength to withstand the imposed loads. Thus 12 gage steel prestressing wire having an ultimate tensile strength of 255,000 psi was used at a spacing of 12 in. on centers horizontally. It was estimated that each wire would be subjected to a maximum tension of about 570 lb to produce the required precompression of the Styrofoam. Thus, a safety factor of approximately 4 was obtained by the use of the 12-gage wire. One end of each wire was displaced 3 ft horizontally from the straight-line generator of the hyperbolic paraboloid, passing through the other end of the wire. The wires were adjusted with a special tensioning device (see Fig. 5) and were clamped with standard A-1 anchor grips.

TABLE 3 — DEFLECTIONS OF SHELL LOADED WITH ¾-IN. SAND ACCORDING TO ESTABLISHED SEQUENCE

Area loaded	Dial gage										
	1	2	3	4	5	6	7	8	9	10	11
1	— 5	40	41	— 2	0	— 5	10	0	3	3	3
1,2	— 10	35	39	— 3	1	— 10	7	— 2	2	2	1
1-3	— 13	35	39	45	37	— 10	7	5	2	2	2
1-4	— 15	35	39	41	36	— 14	11	4	3	2	2
1-4*	— 21	34	42	37	36	— 20	2	1	3	4	3
1-5	— 24	25	40	35	36	— 31	— 8	— 8	2	2	1
1-6	— 29	75	45	31	38	— 12	81	5	5	7	5
1-7	— 33	68	44	29	38	— 23	74	— 2	4	5	3
1-8	— 34	111	50	27	38	— 23	81	— 2	5	6	4
1-9	— 34	108	50	24	37	— 26	79	— 4	5	6	4
1-10	— 30	110	50	79	44	7	104	91	6	9	7
1-11	— 29	107	49	122	50	7	99	92	5	7	5
1-12	— 28	112	50	121	50	5	100	92	6	7	5
1-13	26	97	46	127	50	5	87	87	4	4	3
1-14	101	156	47	127	49	88	129	95	5	5	4
1-15	160	160	46	124	48	94	129	95	4	5	4
1-16	224	158	43	176	47	206	136	140	2	3	4

Note: All deflections are given in thousandths of an inch. Downward deflections are considered positive.
*Deflections taken after load remained on the shell 45 hr.

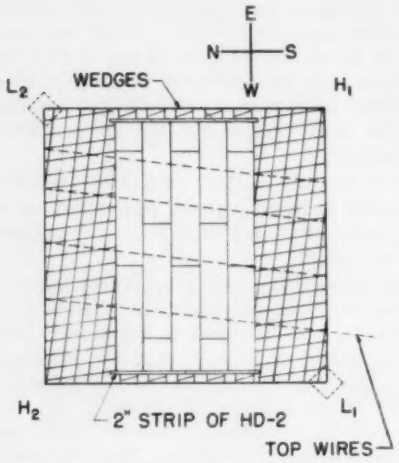

Fig. 6—Arrangement of Styrofoam

Initial adjustment of the wires compensated for an expected sag of the center wires in the amount of 5½ in. due to the placement and precompression of the Styrofoam shell. Proportionately less allowance was made to approach zero allowance for the wires bordering the edge beams.

Before the Styrofoam planks were laid in place, a few of the top wires were put into position to hold the warped planks. The planks were then laid loosely from the center outward toward the edge beams. So as not to have all of the joints in the same line, the Styrofoam planks were laid in brick fashion as shown in Fig. 6. The flat planks of Styrofoam, having an appreciable width, had to be warped through a considerable angle of twist. It was desirable that each plank receive its proper twist and that continuity of the true warped surface exist between adjoining planks. To accomplish this, sheet metal clips having H- and Z-sections were fabricated. Two Z-sections were placed on each end of each plank and two H-sections were placed along the side.

On placement of a length of Styrofoam across the complete length of the shell, two wedges were placed between each edge beam and the Styrofoam. A 2-in. wide strip of HD-2 Styrofoam was placed along the edge of the less dense Styrofoam of the main shell surface to prevent the wedges — which were also made of HD-2 — from cutting into the surface. Fig. 7 shows a close-up of an edge strip and wedge combination. The wedges were oriented so as to be tightened by shearing forces expected to act parallel to each edge beam toward the lower corners of the shell. After all of the Styrofoam shell was laid, edge strips and wedges were placed on the other two sides. In general, the wedges fit well between the steel edge beams and the Styrofoam; however,

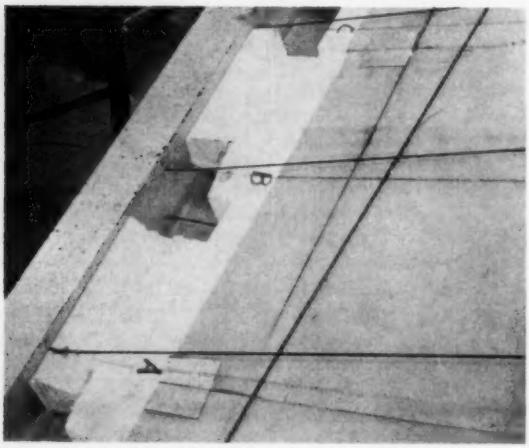

Fig. 7—Wedges in position

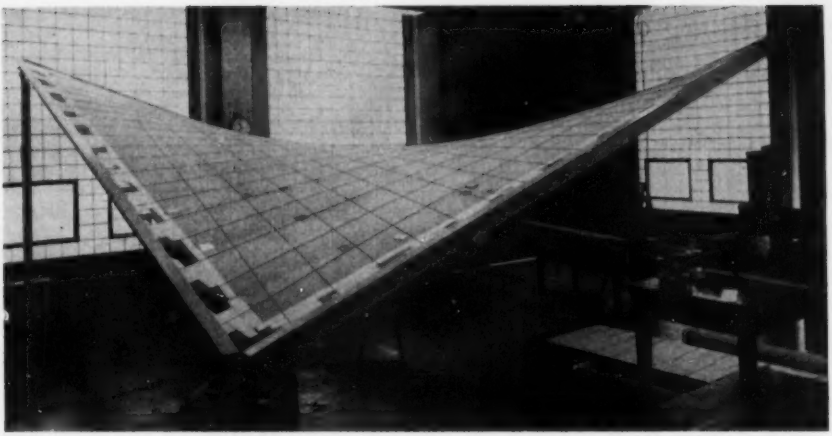

Fig. 8—Close-up elevation from low corner

near the high and low corners of the surface where the slopes of the shell were great, the base of each wedge had to be cut on a slope to fit the edge beam.

The placement of the wedges was followed by the completion of the top wire lattice. Each of the top wires was skewed two spaces from the straight-line generator. The top and bottom wires were skewed in opposite directions so that each plank was confined between the two wire lattices in its twisted position as a part of the hyperbolic paraboloid surface.

Final tightening of the wedges was accomplished with a pipe clamp similar to those used in furniture making. The clamp proved to be quite efficient for

TABLE 4 — DEFLECTIONS OF SHELL UNDER PARTIAL LOADING—4.8 PSF

Area loaded	Deflections, in.*										
	Deflectometer No.										
	1	2	3	4	5	6	7	8	9	10	11
I	0.11	0.08	0.01	—0.04	0.00	—0.01	—0.08	—0.11	0.00	—0.06	—0.10
I-II	0.12	0.08	0.04	—0.08	0.01	0.12	—0.10	—0.22	0.23	0.05	—0.11
I-III	0.14	0.31	0.30	0.02	0.01	0.26	—0.12	—0.32	0.26	0.00	—0.14
I-IV	0.12	0.28	0.29	0.08	0.01	0.32	0.19	0.10	0.35	0.37	0.19
I-V	0.07	0.37	0.38	0.54	0.01	0.41	0.59	0.58	0.36	0.48	0.53
I-III†	0.22	0.74	0.68	0.64	0.02	0.67	0.42	0.37	0.62	0.55	0.47
	Deflectometer No.										
	12	13	14	15	16	17	18	19	20	21	22
I	—0.06	0.04	0.08	0.00	—0.02	0.00	0.12	0.00	0.12	0.00	0.02
I-II	—0.11	0.26	0.07	0.04	0.12	0.24	0.12	0.00	0.14	0.01	0.02
I-III	0.00	0.25	0.29	0.23	0.20	0.25	0.12	0.01	0.14	0.01	0.02
I-IV	0.08	0.37	0.28	0.25	0.29	0.34	0.12	0.01	0.11	0.01	0.02
I-V	0.49	0.35	0.36	0.34	0.36	0.34	0.08	0.01	0.07	0.01	0.00
I-III†	0.53	0.65	0.66	0.58	0.60	0.62	0.20	0.04	0.22	0.01	0.00

*Minus indicates upward deflection.
†Loaded with an additional 4.8 psf following complete loading.

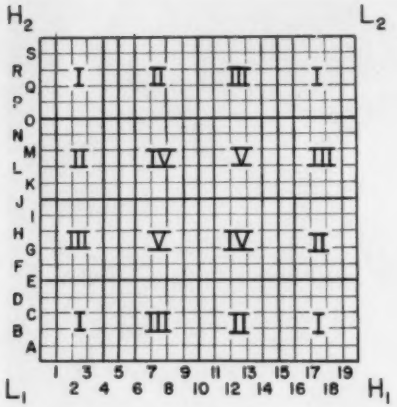

Fig. 9—Surface grid

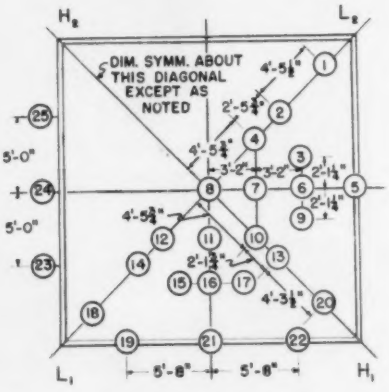

Fig. 10 — Location of deflectometers

this purpose although it lacked a means of measuring the amount of compressive force applied through the wedges. To compress all of the wedges equally, one had to rely on judgment; however this method proved to be quite adequate. The compression in the wedges and the tension in the wires were adjusted until adequate precompression was obtained and the Styrofoam was approximately on the desired surface. On the average, the bottom wires were pretensioned with about 600 lb in each wire. The top wires carried about 200 lb each.

The final precompression of the Styrofoam thus completed the construction of the shell. Fig. 8 shows the completed shell.

Accuracy of construction

To determine the accuracy of the construction procedure, the developed surface was checked to determine the amount of deviation of the surface from the true hyperbolic paraboloid. For this purpose and for use during later testing, grid lines were marked on the top surface of the shell. Each element of the area between horizontally orthogonal grid lines measured 1 sq ft on the horizontal projection. Fig. 9 shows the grid layout and identifies each grid line. The deviations were measured at 2-ft intervals along Grid Lines 5, 10, and 15; and E, J, and O. The maximum deviation of the constructed surface from the true hyperbolic paraboloid surface was found to be ½ in. Thus it is seen that the construction procedure lends itself to good accuracy with proper care.

Apparatus

To be able to compare the behavior of this laboratory shell with the small model, deflectometers were connected at points corresponding to those of the small model. Additional deflectometers were also provided to serve as a check of stations located symmetrically with respect to the major axis through the high points of the hyperbolic paraboloid. The location of the deflectometers is shown on Fig. 10. Each deflectometer consisted of a segment of level-rod scale mounted on a wood strip. Each deflectometer was suspended by a wire from an eyebolt welded to a small steel plate which, in turn, was cemented to the under surface of the Styrofoam shell with epoxy resin. Thus, vertical movements of the shell could be observed with a surveyor's level. Similar segments of level-rod scale were fastened horizontally to an edge beam to

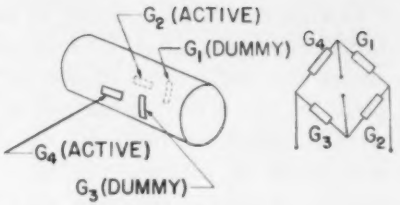

Fig. 11 — Arrangement of SR-4 strain gages on tie rod

observe its horizontal deflections. These were connected on the bottom flange of the Z-section at quarter points and at midlength of the edge beam. A transit was used to observe the deflections of the edge beam.

A slide indicator was mounted on the base of the abutment while a plumb bob was suspended from an attachment near the top of the abutment to measure rotation. A single scale for both was fixed on the floor. A similar arrangement was used to check the horizontal movement of the high corners. The plumb bob scale in this case was two-directional to measure resultant horizontal movement in any direction. No vertical movement of the high corners could be observed due to the fact that the pipe columns were in place during testing.

Four SR-4 electrical resistance strain gages were cemented onto the tie rod near one of the abutments to measure the horizontal thrust resisted by the tie rod. To provide temperature compensation and to eliminate the effect of bending strains in the tie rod, the gages were mounted as shown in Fig. 11. The wiring arrangement also increased the sensitivity of the strain gages through additional output from the transverse gages. The tie rod and its strain gage instrumentation were calibrated in a tensile testing machine to obtain an accurate relationship between the measured strains and applied loads.

Loading

The sequence followed in loading the laboratory shell was the one established during tests on the small laboratory model. In Fig. 9 the Roman numerals denote the loading sequence of subareas of the shell. Regular bricks weighing 4.8 lb each were used as a loading material. The 1-ft horizontal grid spacing served as a guide in placing the bricks uniformly.

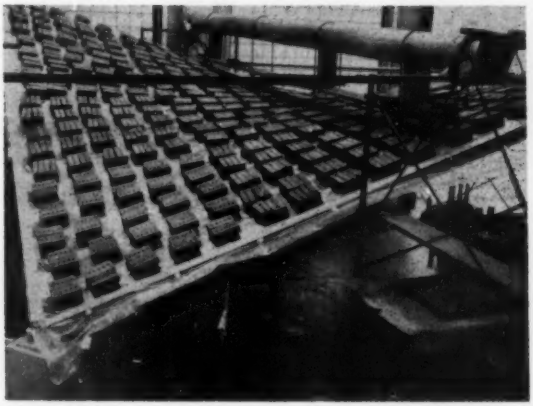

Fig. 12—Shell loaded with 19.2 psf dead load

During the initial loading of the shell with 1 brick per sq ft, deflections were read after loading each subarea in the sequence. Table 4 shows the deflections caused by these partial loadings. After a complete loading with 1 brick per sq ft Subareas I, II, and III were loaded with an additional brick per square foot. This additional partial loading was made to indicate the behavior of the shell if subjected to the weight of plastic concrete with variable thickness, the thickness increasing from a minimum in the center to a maximum at the edges. The deflections for this loading are given in the last line of Table 4.

Following the established sequence, the shell was then loaded uniformly with totals of 9.6, 14.4, and 19.2 psf. The corresponding deflections are recorded in Table 5. Fig. 12 shows the Styrofoam shell completely loaded with 19.2 psf. The curvature of the shell when fully loaded was quite regular and free of buckling tendencies.

TABLE 5 — DEFLECTIONS OF SHELL LOADED UNIFORMLY

Load, psf	Deflections, in.*										
	Deflectometer No.										
	1	2	3	4	5	6	7	8	9	10	11
4.8	0.07	0.37	0.38	0.54	0.01	0.41	0.59	0.58	0.36	0.48	0.53
9.6	0.16	0.83	0.80	1.08	0.01	0.83	1.15	1.16	0.73	1.01	1.12
14.4	0.29	1.37	1.31	1.85	0.02	1.34	1.78	1.94	1.20	1.67	1.92
19.2	0.38	1.88	1.85	2.60	0.04	1.90	2.66	2.70	1.66	2.28	2.60
	Deflectometer No.										
	12	13	14	15	16	17	18	19	20	21	22
4.8	0.49	0.35	0.36	0.34	0.36	0.34	0.08	01	0.07	0.01	0.00
9.6	1.02	0.77	0.72	0.68	0.74	0.72	0.14	0.04	0.16	0.01	0.00
14.4	1.74	1.28	1.20	1.14	1.22	1.19	0.29	0.08	0.29	0.04	0.00
19.2	2.40	1.74	1.66	1.58	1.69	1.66	0.37	0.10	0.40	0.05	0.00

*Minus indicates upward deflection.

TABLE 6 — EDGE BEAM DEFLECTION, TIE ROD TENSION, AND HIGH CORNER MOVEMENT

Load, psf	Beam deflection, in.*			Tie rod tension, lb	High corner movement, in.†	
	Deflectometer No.					
	23	24	25		H_{1-x}	H_{1-y}
4.8	0.02	0.01	0.00	1950	—.05	.05
9.6	0.05	—0.01	—0.04	3750	.00	.00
14.4	0.06	—0.04	—0.07	5500	.10	—.05
19.2	0.07	—0.06	—0.14	7150	.10	—.05

*Inward movement of the beam is considered negative.

† H_{1-x} designates movement in x-direction and is considered negative if movement is toward L_1. H_{1-y} designates movement in the y-direction and is considered negative if movement is toward L_2.

In addition to the measurement of vertical deflections of the shell during loading, data pertinent to the deflections of the edge beams and shell supports as well as the forces induced in the tie rod were compiled. These data are shown in Table 6. No sliding of the abutments was observed and the rotation of the abutments was, for all practical purposes, negligible. The two horizontal components of movement of one high corner are also given in Table 6.

To check for possible creep, the 19.2 psf uniform load was left on the shell for 27 hr. During that period, the maximum increase in deflection was noted to be about ¼ in. at Deflectometer Station 4. Since this increase was so small, it does not seem that creep deflections would be an important factor.

Since further tests were to be conducted on the shell, it was not tested to destruction. The maximum uniform load of 19.2 psf was believed to be representative of a satisfactory thickness of permanent concrete cover for this size of shell.

STIFFENING OF THE STYROFOAM SHELL

For experiments leading to further stiffening of the Styrofoam shell, a mortar consisting of one part high-early-strength cement and three parts sand (by weight) was troweled onto the upper surface of the shell. The thickness of this crust varied from about ⅛ in. to something greater than ¼ in. The total weight of the material used indicated an average thickness of 0.22 in. This mortar was cured under a cover of polyethylene film which retained the moisture in or near the curing crust. This crust contained no effective reinforcement. Although it essentially covered the upper lattice of wires, the wires were not oriented as effective tensile or compressive reinforcement for the crust of mortar.

After the mortar was cured, the coated shell was again subjected to partial and complete uniform loading. Deflection data were again collected and are tabulated in Table 7. A comparison of the data in Tables 4 and 5 with that of Table 7 shows the effectiveness of the mortar crust in reducing deflections. For example, the maximum deflection (at the center) of the uncoated Styrofoam shell when subjected to a total uniform load of 19.2 psf was 2.70 in. In the coated shell this maximum deflection was reduced to 0.43 in. Such reductions were observed throughout the shell and it can be seen that the crust thus stiffened the shell by a factor of about 6.

The mortar-covered Styrofoam shell was subjected to a total uniform load of 24.1 psf. At a load of 19.2 psf no cracks could be observed in the cover. However, at the maximum load of 24.1 psf, one main crack traversed the shell in a direction approximately at right angles to the

TABLE 7 — DEFLECTIONS OF COATED SHELL UNDER PARTIAL AND UNIFORM LOADING

Area loaded	Deflections, in.*										
	Deflectometer No.										
	1	2	3	4	5	6	7	8	9	10	11
I-III 4.8 psf	0.07	0.10	0.06	0.05	0.00	0.04	0.00	—0.04	0.05	0.01	—0.02
I-V 4.8 psf	0.04	0.11	0.08	0.08	0.01	0.07	0.08	0.07	0.08	0.07	0.05
I-V 19.2 psf	0.01	0.26	0.32	0.37	0.05	0.34	0.38	0.43	0.35	0.38	0.42
I-V 24.1 psf	—0.01	0.36	0.44	0.55	0.08	0.48	0.60	0.65	0.50	0.58	0.62
	Deflectometer No.										
	12	13	14	15	16	17	18	19	20	21	22
I-III 4.8 psf	0.01	0.06	0.07	0.05	0.05	0.06	0.05	0.01	0.07	0.01	0.01
I-V 4.8 psf	0.07	0.08	0.06	0.06	0.07	0.06	0.00	0.00	0.05	0.01	0.01
I-V 19.2 psf	0.46	0.24	0.28	0.34	0.34	0.34	0.02	0.05	0.14	0.02	0.02
I-V 24.1 psf	0.60	0.53	0.36	0.41	0.46	0.48	0.05	0.07	0.22	0.06	0.06

*Minus indicates upward deflection.

direction of the principal tensile stresses. Based on the average thickness of the shell, the average tensile stress in the crust, if it were considered to be carrying the total superimposed load, would have been about 130 psi.

RESULTS OF LABORATORY MODEL STUDIES

General discussion

The results obtained from experiments on the small-scale construction model and the larger laboratory construction model indicate that the construction procedure studied would warrant trial in the field. During the construction of the laboratory shell no major problems were encountered. The deviations from the true surface were quite small, the maximum being about ½ in. Through further adjustment of the wires, the deviation could be reduced, if necessary.

The flexibility of the individual Styrofoam planks indicates an adaptability to this type of construction. Even though nearly the entire area of the shell was constructed from full rectangular planks, there were no objectionable gaps between the individual planks. This is attributed to the system of precompressing the Styrofoam, which closed the gaps and caused the shell to act as a thin-shell membrane.

A minor problem that was encountered during the precompression of the Styrofoam was the deformation of the wedges. It is believed that more slender wedges requiring less driving force would cure this difficulty.

Comparison of deflections in large and small models

In comparing the deflections in the small model with those of the large model one can make use of Reissner's deflection formula.[4] This formula, based on the assumed boundary condition of no horizontal movement at the edges of the membrane, is

$$w(0,0) - w(a,b) = \frac{2q}{Gt}\,\frac{(ab)^2}{(2h)^2}$$

This formula gives the magnitude of the vertical deflection of the center of the shell with respect to a corner. Thus the deflection of the center with respect to the high corners at which the deflection was held to zero would be

$$w = \frac{2q}{Gt}\,\frac{(ab)^2}{(2h)^2}$$

where

q = the load on a unit horizontal projection of the surface
a = half-width of rectangular hyperbolic paraboloid in the X-direction
b = half-width of rectangular hyperbolic paraboloid in the Y-direction
G = modulus of rigidity
t = thickness of shell surface
h = half-rise of edge beam of hyperbolic paraboloid

Thus the ratio of the deflections of the small and large models m and p, respectively, made of the same material, is given by

$$\frac{w_m}{w_p} = \frac{(q_m)}{(q_p)}\,\frac{(t_p)}{(t_m)}\,\frac{(a_m b_m)^2}{(a_p b_p)^2}\,\frac{(h_p)^2}{(h_m)^2}$$

Substitution for the appropriate constants a, b, h, and t which were applicable to the small and large models yields the ratio

$$\frac{w_m}{w_p} = 0.263\,\frac{q_m}{q_p}$$

For the maximum loads applied to the Styrofoam shells, i.e., $q_m = 16.70$ psf and $q_p = 19.2$ psf, the theoretical ratio of the maximum deflections is

$$\frac{w_m}{w_p} = 0.229$$

From the experimental data for the center deflections the observed ratio was

$$\frac{w_m}{w_p} = \frac{0.689}{2.70} = 0.255$$

This indicates that the deflection of the large model was relatively smaller than that of the small model. For loads other than the maximum loading, the observed ratios w_m/w_p were also a little larger than

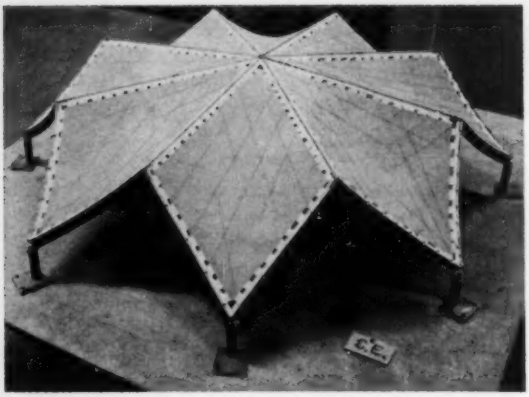

Fig. 13—Construction model of spherical type shell of hyperbolic paraboloids

the computed ratio. Some of these differences can be attributed to the relative flexibility of the wire adjustment system in the small model as compared to the laboratory model system. However, it appears that from the behavior of a model, a fairly good idea can be obtained as to the behavior of the actual structure.

Horizontal thrust in the tie rod

The tie rod strain gage system was used to obtain the magnitude of the thrust resisted by the tie rod. When the shell was subjected to a uniform load of 19.2 psf, the tie rod load was 7150 lb. This corresponds to the theoretical value of abutment thrust of 7760 lb. The difference, which is less than 8 percent, is most likely attributable to the rigidity of the edge beam connections at the abutments and to some resistance to thrust developed by the abutment on the floor.

EXPERIMENTS WITH OTHER THIN-SHELL SHAPES

The general construction procedure developed and tested through laboratory experiments also shows considerable promise for shapes other than the square hyperbolic paraboloid. Experiments are being conducted on a spherical type shell made up of several truncated,

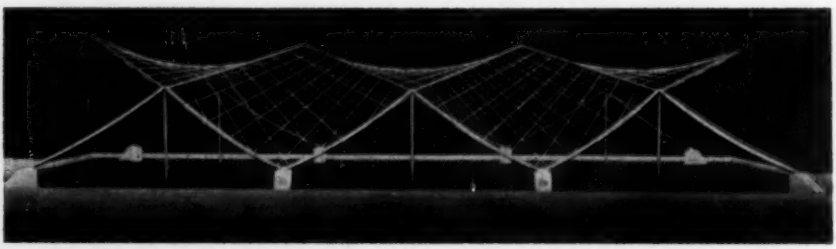

Fig. 14—Space model of barrel type shell of hyperbolic paraboloids

skewed hyperbolic paraboloids. A table sized construction model of such a shell is shown in Fig. 13. The fundamental geometric considerations which make the proposed construction system possible are valid for this type of shape. Further studies are to be conducted on this shell type.

Barrel type shells made up of full, skewed hyperbolic paraboloids in conjunction with truncated, skewed hyperbolic paraboloids seem also to be a feasible application of the general construction procedure being studied. Fig. 14 shows a space model of such a shell structure. This type of shell structure is also the subject of further studies.

CONCLUDING REMARKS

The research program has consisted of studies of a new method of forming thin-shell structures and has led to the development of new combinations of hyperbolic paraboloids. Three types of models have been used to advantage in these studies; namely, space models, table sized construction models, and laboratory sized construction models.

Space models similar to that shown in Fig. 14 for the barrel type shell have proved to be a great aid in the early stages of planning such shell structures. These space models, with elastic strings stretched on generators of the hyperbolic paraboloids facilitate the visualization of the external curvatures of the shells, allow easy measurement of all dimensions, and provide for experiments with internal layouts.

Table sized construction models similar to those of Fig. 2 and 13 are of special value in establishing construction procedures, and predicting construction difficulties as well as indicating mechanical response of the prototype when subjected to loads.

The laboratory construction model is but one step away from a full-scale structure. Undoubtedly the results of tests on the large models are valid predictors of the action of a similar but larger field structure at the same stage of completion. Thus the success of the laboratory experiments indicates that field constructions using the same materials and the outlined procedure are sound and may prove more economical, after a number of applications, than the present practice involving extensive falsework. Perhaps the method may prove most beneficial, in the case of large shell constructions, in forming the surfaces and by greatly reducing the amount of falsework required rather than by eliminating falsework completely. Further developments and improvements could be expected to come from application of the procedure in field constructions.

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For the sake of brevity, many details concerning the experiments have been omitted and may be found in a Purdue University thesis.[7]

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Title No. 57-21

Determination of Membrane Stresses in Elliptic Paraboloids Using Polynomials

By L. FISCHER

A mathematically simple method for determining the membrane stresses in an elliptic paraboloid is given. The differential equations of equilibrium are solved by the introduction of a polynomial representing the stress function, the coefficients of which are easily determined. The membrane stresses are then readily found as derivatives of the stress function. A numerical example illustrates the use of this method.

■ TRANSLATIONAL SHELLS, SUPPORTED ALONG THEIR FOUR EDGES by mutually perpendicular arcs, are analyzed. Such shells carry their own weight and superimposed load mainly by membrane stresses without bending. Mathematical difficulties have so far kept many engineers from using these shells, which otherwise show clear advantages. In the following an effort is made to overcome these difficulties by introducing a simple method of determining the membrane stresses.

In Fig. 1 an elliptic paraboloid is shown. The shell surface is formed by translation of a parabolic arc P_1 along a parabola P_2 or vice versa. The paraboloid is termed "elliptic" because all horizontal contour lines of the surface form ellipses. An orthogonal coordinate system is defined in Fig. 1.

The equations of the parabolas are:

$$(P_1) \qquad z_1 = \frac{f_1}{a^2}\, x^2$$

$$(P_2) \qquad z_2 = \frac{f_2}{b^2}\, y^2$$

If the parabolas are flat, they may be replaced by circles whose radius is given by:

$$R_1 = 1 : \frac{d^2 z_1}{dx^2} = \frac{a^2}{2f_1}$$

$$R_2 = 1 : \frac{d^2 z_2}{dy^2} = \frac{b^2}{2f_2}$$

L. Fischer is associate professor of structural engineering, Technion-Israel Institute of Technology, Haifa, Israel. Dr. Fischer received his education and worked as a consulting engineer in Czechoslovakia before moving to Israel in 1949. He engaged extensively in the design of concrete shell structures before receiving his present appointment in 1957. He currently teaches graduate courses in shell design.

The equation of the shell surface is then:

$$z = z_1 + z_2 = \frac{f_1 x^2}{a^2} + \frac{f_2 y^2}{b^2} = \frac{x^2}{2R_1} + \frac{y^2}{2R_2}$$

For simplicity let us assume a square shell formed by identical parabolas, i.e.,

$$a = b$$

$$f_1 = f_2 = f$$

and therefore also

$$R_1 = R_2 = R$$

The equation of the shell surface then becomes:

$$z = \frac{1}{2R}(x^2 + y^2) \qquad (1)$$

Membrane stresses

Fig. 2 shows an element of the shell and the membrane stresses N_x, N_y, and T acting on it. The solution is simplified by finding first the horizontal projection of the stresses, i.e., n_x, n_y, and t, respectively, and then expressing the actual stresses by them.

Without giving the derivation, the actual stress is related to its horizontal projection in the following way:

$$\left.\begin{aligned} N_x = n_x \frac{\cos \psi}{\cos \phi} \qquad N_y = n_y \frac{\cos \phi}{\cos \psi} \qquad T = t \\ \tan \phi = \frac{\partial z}{\partial x} \qquad \tan \psi = \frac{\partial z}{\partial y} \end{aligned}\right\} \qquad (2a)$$

The problem of determining the membrane stresses can be reduced to finding the stress function $F(x,y)$ that will satisfy the following equation:

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = -Rq_z \qquad (2b)$$

q_z denotes the load per unit of projected area in direction z. If the shell is subjected only to its own dead load:

$$q_z = g\sqrt{1 + \tan^2 \phi + \tan^2 \psi} \qquad (2c)$$

where g is the weight of a unit area of the shell surface. If d denotes

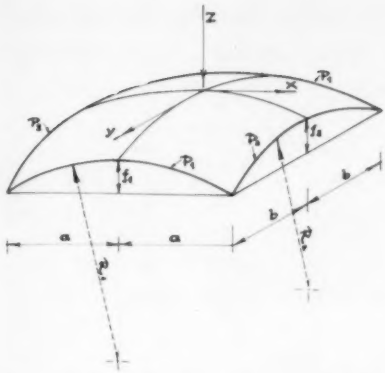

Fig. 1—Coordinate system and dimensions of an elliptical paraboloid

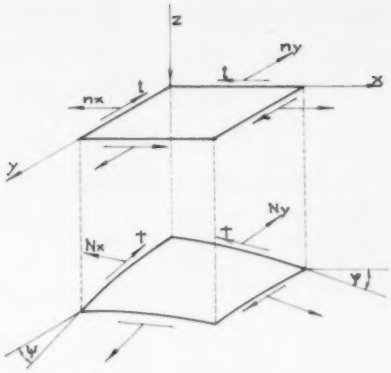

Fig. 2—Membrane stresses in a shell element

the shell thickness and γ its specific weight, the load per unit area of shell surface will be:

$$g = d\gamma$$

After determining the function $F(x,y)$ that will satisfy Eq. (2b), the projected membrane stresses are given by:

$$n_x = \frac{\partial^2 F}{\partial y^2} \qquad n_y = \frac{\partial^2 F}{\partial x^2} \qquad t = -\frac{\partial^2 F}{\partial x \partial y} \qquad \text{(2d)}$$

The true membrane stresses are then found by the relations given in Eq. (2a).

Determination of stress function *F* and membrane stresses

The load on the shell, Eq. (2c), can be rewritten as a polynomial in x and y. To do so, we put

$$\tan \phi = \frac{\partial z}{\partial x} = \frac{x}{R} \qquad \text{and} \qquad \tan \psi = \frac{\partial z}{\partial y} = \frac{y}{R}$$

Substituting these values into Eq. (2c) and developing it by Newton's binomial, we get:

$$q_z = g\left(1 + \frac{x^2 + y^2}{R^2}\right)^{1/2}$$

$$= g\left[1 + \frac{1}{2R^2}(x^2 + y^2) - \frac{1}{8R^4}(x^4 + y^4) - \frac{1}{4R^4}x^2y^2\right] \qquad \text{(3a)}$$

The function F can be expressed in the general form:

$$F = A(x^2 + y^2) + Bx^2y^2 + C(x^4 + y^4) + D(x^6 + y^6) + E(x^4y^2 + x^2y^4)$$
$$+ Fx^4y^4 + G(x^6y^2 + x^2y^6) \qquad \text{(3b)}$$

Substituting Eq. (3a) and (3b) and derivatives into Eq. (2b), we get:

$$(2A + 2By^2 + 12Cx^2 + 30Dx^4 + 12Ex^2y^2 + 2Ey^4 + 12Fx^2y^4 + 2Gy^6 + 30Gx^4y^2) + (2A + 2Bx^2 + 12Cy^2 + 30Dy^4 + 12Ey^2x^2 + 2Ex^4 + 12Fy^2x^4 + 2Gx^6 + 30Gx^2y^4) = -gR\left[1 + \frac{1}{2R^2}(x^2 + y^2) - \frac{1}{8R^4}(x^4 + y^4) - \frac{1}{4R^4}x^2y^2\right]$$

To obtain the unknown coefficients $A, B, \ldots G$, we can equate the coefficients of variables with the same exponent. Thus the following identities are established:

$$\left.\begin{aligned} 2A + 2A + gR &= 0 \quad \text{(a)} \\ (x^2 + y^2)\left(2B + 12C + \frac{g}{2R}\right) &= 0 \quad \text{(b)} \\ x^2y^2\left(12E + 12E - \frac{g}{4R^3}\right) &= 0 \quad \text{(c)} \\ (x^4 + y^4)\left(30D + 2E - \frac{g}{8R^3}\right) &= 0 \quad \text{(d)} \end{aligned}\right\} \quad \text{(3c)}$$

So far only four equations, Eq. (3c), are set up; the remaining three necessary to determine the seven unknowns can be found from the boundary conditions. Since the arcs along the boundaries have no stiffness perpendicular to their plane, the following conditions are established:

$$\left.\begin{aligned} \text{for } x = \pm a \quad & n_x = 0 \\ \text{for } y = \pm a \quad & n_y = 0 \end{aligned}\right\} \quad \text{(3d)}$$

The membrane stresses are expressed in terms of the derivatives of Eq. (3b) introduced into Eq. (2d), which yield:

$$\left.\begin{aligned} n_x = \frac{\partial^2 F}{\partial y^2} &= 2A + 2Bx^2 + 12Cy^2 + 30Dy^4 + E(12y^2x^2 + 2x^4) + 12Fy^2x^4 + G(2x^6 + 30x^2y^4) \\ n_y = \frac{\partial^2 F}{\partial x^2} &= 2A + 2By^2 + 12Cx^2 + 30Dx^4 + E(12x^2y^2 + 2y^4) + 12Fx^2y^4 + G(2y^6 + 30x^4y^2) \\ t = \frac{\partial^2 F}{\partial x \partial y} &= -4Bxy - 8E(x^3y + y^3x) - 16Fx^3y^3 - 12G(xy^5 + x^5y) \end{aligned}\right\} \quad \text{(3e)}$$

Substituting Eq. (3e) into the first boundary condition given by Eq. (3d), we get:

$$n_x(x = \pm a) = 0$$

$$2A + 2Ba^2 + 12Cy^2 + 30Dy^4 + E(12a^2y^2 + 2a^4) + 12Fa^4y^2 + G(2a^6 + 30a^2y^4) = 0$$

This again, being an identity, permits us to gain three additional equa-

tions for the determination of the coefficients of the stress function, namely:

$$\left.\begin{aligned} 2A + 2Ba^2 + 2Ea^4 + 2Ga^6 &= 0 \quad \text{(a)} \\ y^2(12C + 12Ea^2 + 12Fa^4) &= 0 \quad \text{(b)} \\ y^4(30D + 30Ga^2) &= 0 \quad \text{(c)} \end{aligned}\right\} \quad \text{(3f)}$$

When satisfying the boundary condition for n_y, exactly the same equations are obtained. Eq. (3f) and (3g) now are sufficient to determine the seven unknown coefficients A through G of the stress function $F(x,y)$.

$$\left.\begin{aligned} &\text{from Eq. (3c)(a):} \quad A = -\frac{gR}{4} \\ &\text{from Eq. (3c)(c):} \quad E = +\frac{g}{96R^3} \\ &\text{from Eq. (3c)(d):} \quad D = +\frac{g}{288R^3} \\ &\text{from Eq. (3f)(c):} \quad G = -\frac{g}{288R^3a^3} = -\frac{g}{288m^3R^6} \\ &\text{from Eq. (3f)(a):} \quad B = +\frac{g}{144m^2R}(36 - m^4) \doteq +\frac{g}{4m^2R} \\ &\text{from Eq. (3c)(b):} \quad C \doteq -\frac{g}{24Rm^2}(1 + m^2) \\ &\text{from Eq. (3f)(c):} \quad F = +\frac{gR}{24a^6}\left(1 + m^2 - \frac{m^4}{4}\right) \doteq +\frac{gR}{24a^6}(1 + m^2) \end{aligned}\right\} \quad \text{(3g)}$$

In Eq. (3g) we have substituted $m = a/R$. In general, $f \doteq a/10$ and therefore

$$m = \frac{a}{R} = \frac{a}{\frac{a^2}{2f}} = \frac{2f}{a} \doteq 0.2$$

consequently m^4 may safely be neglected, as was done above.

Substituting the coefficients from Eq. (3g) into Eq. (3c) we finally obtain the solution for the membrane stresses:

$$n_x = -\frac{gR}{2} + \frac{gx^3}{2Rm^2} - \frac{gy^2}{2Rm^2}(1 + m^2) + \frac{5y^4}{48R^3} + \frac{gx^4}{48R^3} + \frac{gx^2y^2}{8R^3} + \frac{gx^4y^2}{2R^5m^6}(1 + m^2) - \frac{g}{144m^2R^6}(x^6 + 15x^2y^4)$$

or

$$n_x = -\frac{gR}{2}\left[1 - \frac{x^2}{a^2} + \frac{y^2}{a^2}(1 + m^2) - \frac{5y^4m^4}{24a^4} - \frac{x^4m^4}{24a^4} - \frac{x^2y^2m^4}{4a^4} - \frac{x^4y^2}{a^6}(1 + m^2) + \frac{x^6 + 15x^2y^4}{72a^6}m^4\right]$$

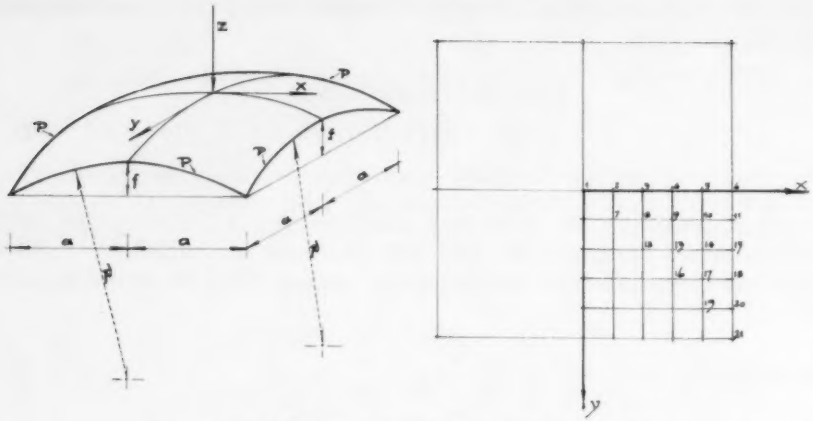

Fig. 3—(a) Elliptical paraboloid. (b) Basic grid

Since products of m^4 may be neglected, the expressions for the membrane stresses are reduced to:

$$n_x = -\frac{gR}{2}\left[1 - \frac{x^2}{a^2} + \frac{y^2}{a^2}(1 + m^2) - \frac{x^4y^2}{a^6}(1 + m^2)\right]$$

the two other stresses will be given by:

$$n_y = -\frac{gR}{2}\left[1 - \frac{y^2}{a^2} + \frac{x^2}{a^2}(1 + m^2) - \frac{y^4x^2}{a^6}(1 + m^2)\right]$$

$$t = -\frac{gxy}{m^2R} - \frac{gxy}{12R^3}(x^2 + y^2) - \frac{16gR}{24a^6}x^3y^3(1 + m^2) + \frac{gxy}{24m^2R^5}(x^4 + y^4)$$

or

$$t = -\frac{gRxy}{a^2}\left[1 + \frac{2x^2y^2}{3a^4}(1 + m^2) + \frac{x^2 + y^2}{12a^2}m^4 - \frac{x^4 + y^4}{24a^4}m^4\right]$$

Neglecting members with m^4, this becomes:

$$t = -\frac{gRxy}{a^2}\left[1 + \frac{2}{3}\frac{x^2y^2}{a^4}(1 + m^2)\right]$$

Introducing into these expressions

$$\xi = \frac{x}{a} \qquad \eta = \frac{y}{a} \qquad \text{(3h)}$$

the final values of the projected membrane stresses are obtained:

$$\left.\begin{aligned} n_x &= -\frac{gR}{2}\left[1 - \xi^2 + \eta^2(1 + m^2)(1 - \xi^4)\right] \\ n_y &= -\frac{gR}{2}\left[1 - \eta^2 + \xi^2(1 + m^2)(1 - \eta^4)\right] \\ t &= -gR\,\xi\,\eta\left[1 + \frac{2}{3}\xi^2\eta^2(1 + m^2)\right] \end{aligned}\right\} \qquad \text{(3i)}$$

After introducing Eq. (3i) into Eq. (2a) the actual membrane stresses are finally determined.

NUMERICAL EXAMPLE

An elliptic paraboloid whose dimensions are given in Fig. 3 will now be analyzed for the membrane stresses due to its own weight.

Using Eq. (3i) the projected stresses are readily determined, their values are summarized in Columns 4, 5, and 6 of Table 1. Columns 7, 8, and 9 of Table 1 give these stresses for comparison, as determined by the method of finite differences. These values were reproduced here from a numerical example worked out for an identical shell by Brankov.[1]

The following illustrates, for Point 14 of the shell (Fig. 3), how the values in Columns 4, 5, and 6 of Table 1 were obtained. Introducing $m = 0.2$ and

$$\xi = \frac{x}{a} = 0.8 \qquad \eta = \frac{y}{a} = 0.4$$

TABLE I — DETERMINATION OF PROJECTED MEMBRANE STRESSES

1	2	3	4	5	6	7	8	9
Point	ξ	η	$\frac{n_x}{gR}$	$\frac{n_y}{gR}$	$\frac{t}{gR}$	$\frac{n_x}{gR}$	$\frac{n_y}{gR}$	$\frac{t}{gR}$
1	0.0	0.0	—0.500	—0.500	0.000	—0.500	—0.500	0.000
2	0.2	0.0	—0.480	—0.521	0.000	—0.478	—0.522	0.000
3	0.4	0.0	—0.420	—0.583	0.000	—0.415	—0.586	0.000
4	0.6	0.0	—0.320	—0.687	0.000	—0.309	—0.692	0.000
5	0.8	0.0	—0.180	—0.833	0.000	—0.166	—0.834	0.000
6	1.0	0.0	0.000	—1.020	0.000	0.000	—1.000	0.000
7	0.2	0.2	—0.501	—0.501	—0.040	—0.500	—0.500	—0.043
8	0.4	0.2	—0.440	—0.563	—0.080	—0.435	—0.565	—0.086
9	0.6	0.2	—0.338	—0.667	—0.121	—0.327	—0.673	—0.127
10	0.8	0.2	—0.193	—0.812	—0.163	—0.177	—0.823	—0.159
11	1.0	0.2	0.000	—0.999	—0.206	0.000	—1.000	—0.171
12	0.4	0.4	—0.501	—0.501	—0.163	—0.500	—0.500	—0.174
13	0.6	0.4	—0.392	—0.602	—0.250	—0.386	—0.614	—0.262
14	0.8	0.4	—0.229	—0.744	—0.343	—0.217	—0.783	—0.337
15	1.0	0.4	0.000	—0.927	—0.444	0.000	—1.000	—0.368
16	0.6	0.6	—0.483	—0.483	—0.393	—0.500	—0.500	—0.410
17	0.8	0.6	—0.290	—0.609	—0.557	—0.304	—0.696	—0.558
18	1.0	0.6	0.000	—0.772	—0.750	0.000	—1.000	—0.629
19	0.8	0.8	—0.377	—0.377	—0.822	—0.500	—0.500	—0.759
20	1.0	0.8	0.000	—0.487	—1.155	0.000	—1.000	—0.960
21	1.0	1.0	0.000	0.000	—1.693	—0.500	—0.500	—1.531

into Eq. (3i), we get:

$$n_x = -\frac{gR}{2}\left\{1 - (0.8)^2 + (0.4)^2\,[1 + (0.2)^2]\,[1 - (0.8)^4]\right\} = -0.229gR$$

$$n_y = -\frac{gR}{2}\left\{1 - (0.4)^2 + (0.8)^2\,[1 + (0.2)^2]\,[1 - (0.4)^4]\right\} = -0.744gR$$

$$t = -gR \times 0.8 \times 0.4\left\{1 + \frac{2}{3}(0.8)^2\,(0.4)^2\,[1 + (0.2)^2]\right\} = -0.343gR$$

These results are summarized in Columns 4, 5, and 6. The stresses determined at this point by the method of finite differences are given in Columns 7, 8, and 9. They are in fairly close agreement with the results obtained by the method presented here.

SOME CRITICAL REMARKS

a. The boundary conditions, Eq. (3d), are in contradiction with the compatibility equation, Eq. (2b), at the corners of the shell. This is seen when Eq. (2d) is introduced into Eq. (2b), resulting in

$$n_y + n_x = -Rq_x \qquad (5)$$

Eq. (5) cannot be satisfied at the corners, since both n_x and n_y must vanish. Therefore either the boundary conditions of Eq. (3d) *or* the differential equation, Eq. (2b), can be satisfied at the corners, but never both of them simultaneously. Generally, when analyzing the shell by the method of finite differences, the differential equation is satisfied; at the corners, stresses perpendicular to the boundary arcs are obtained. Eq. (2b) or (5) do not hold at the corners, since there the membrane state is disturbed by local bending moments and part of the load is carried by shear. It therefore seems more reasonable to satisfy the boundary conditions, as in the method presented here, and to disregard Eq. (2b) in the corner region of the shell. This explains the considerable discrepancy in this region between the stresses obtained by the polynomial approach and those obtained by the method of finite differences.

b. The method introduced here is easily adaptable to other shell forms (of positive curvature) and to different conditions of loading.

c. Csonka,[2] in a paper on shells curved in two directions, suggests a different approach towards the determination of membrane stresses in calotte shells. In his method, that of the "indefinite form," he does not start from some shell of definite shape, but from a shell in the $f(x,y)$ function of which, characterizing the middle surface, there are one or several freely eligible parameter values. To the free parameters figuring in the shape function f and the stress function F, such values

are subsequently attributed that Eq. (2b) should yield, at some selected points of the shell, load values in conformity with those given.

Such approach, however, does not seem necessary by the method suggested here; with which a shell of any given shape with any arbitrary loading can be readily analyzed.

d. Another approach toward the solution of stresses in an elliptical paraboloid is presented by Parme.[3] He expresses the stress function by an infinite series, which satisfies the basic differential equation, Eq. (2b). The solution requires the summation of several terms of a series, which in some cases may only slowly converge. The method becomes even more involved when a nonuniform load is applied to the shell. In that case the load function is expressed in terms of a double Fourier series, which makes the algebraic operations quite extensive and formidable.

The numerical example given in Parme's paper, when solved by the author's method, requires considerably less numerical work. The results obtained by that method are in close agreement with those given by Parme, who arrives at about the same values as by the method of finite differences.

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Title No. 57-22

Resistance to Shear of Reinforced Concrete Beams

Part 3 — Beams with Bent-Up Bars

By A. M. NEVILLE and J. TAUB*

The fallacy of the truss analogy in the design of web reinforcement consisting of bent-up bars is discussed. It is shown that after the bent-up bars have yielded, a redistribution of internal forces takes place and the beam can take a considerable increase in load before failure occurs.

The spacing of bent-up bars is discussed, and for full protection from shear failure a limiting value of $2d(1-k)$, measured along the axis of the beam, is suggested. The influence of the cross-sectional area of bent-up bars and the contribution of beam elements to its shear resistance are studied.

From tests on beams subjected to uniformly distributed and nonsymmetrical loads, it is concluded that the provision of bent-up bars in zones of maximum shear only is inadequate. The influence on shear strength of anchorage and cutoff of the tension steel are mentioned.

■ WHERE SHEARING STRESS IN A PART of a reinforced concrete beam exceeds the value allowed by the appropriate code, shear reinforcement has to be provided. This may be in the form of bent-up bars or stirrups or both. In this part of the paper the use of bent-up bars or stirrups inclined at 45 deg is considered, Part 4 will deal with the use of both bent-up bars and vertical stirrups and also with the use of stirrups inclined at angles other than 45 deg to the axis of the beam.

There exist some basic differences between the various codes in their approach to shear reinforcement, and this is particularly so with respect to the use of bent-up bars. ACI 318[33]† and the British code[34] require web reinforcement only in that part of the beam in which the shearing stress exceeds the allowable value, while the German code[35] prescribes

*Authors' biographical information appeared in Part 1, Aug., 1960, ACI JOURNAL, to which it should be added that Dr. Neville has been appointed head of the Department of Civil Engineering, Nigerian College of Technology, Zaria, Nigeria.

†References are numbered consecutively beginning with Part 1 of this five-part series.

Notation

a = shear span
B = force in bent-up bar
C = resultant (inclined) compression force in concrete
C' = horizontal component of the compression force in concrete
d = effective depth of beam
f_c' = compressive strength of concrete test cylinder
f_{cu} = compressive strength of concrete test tube
f_s = tensile stress in main tension reinforcement
f_w = tensile stress in web reinforcement
jd = lever arm of internal forces in beam
kd = depth of compression zone of concrete
P_u = total load on beam at failure
p = percentage tension steel area
r = percentage web reinforcement area
T = force in main tension reinforcement
T_h = horizontal component of T
T_v = vertical component of T
V = total shearing force
V_c = shearing force resisted by concrete above neutral axis
V_p = shearing force carried by concrete at permissible stress
V_u = total shearing force at failure
v = nominal shearing stress in concrete
v_u = nominal shearing stress at failure

web reinforcement throughout the length of the beam. There is, however, a major difference between the ACI and the British codes: the latter requires the web reinforcement to carry the whole shearing force on the section while the former assumes a part of this force to be carried by the concrete.

When bent-up bars are used as the sole web reinforcement their spacing measured along the axis of the beam is prescribed by ACI 318 as not more than the effective depth of the beam when the nominal shearing stress does not exceed $0.06\ f_c'$, and half that distance when the nominal shearing stress is larger. The British code does not explicitly specify a maximum distance between bent-up bars, their spacing being calculated on the assumption that they form the tension web members of one or more single systems of lattice girders in which the concrete forms the compression web members. The German code requires that the spacing of bent-up bars be based on the force which they can carry without exceeding the permissible stress, with the proviso that an imaginary line drawn normal to the bent-up bar at any point along it, crosses at least one bent-up bar over the depth of the beam.

A further difference between the codes concerns specific demands for the use of bent-up bars. As is well known, the ACI code requires that bent-up bars or inclined stirrups be provided when the nominal shearing stress exceeds $0.08\ f_c'$ or 240 psi. The British and German codes do not prescribe any mandatory use of bent-up bars but the German code recommends that they be used to resist the major part of the shearing force on the beam. The British code does not seem to

differentiate between the advantages of the various methods of providing web reinforcement.

DATA ON USE OF BENT-UP BARS AND TRUSS ANALOGY

Some valuable data on the strength in shear of rectangular reinforced concrete beams whose web reinforcement consists of bent-up bars have been obtained by Moretto[31] and more recently by Moody, Viest, Elstner, and Hognestad[3] in the United States, and by Wilby[17] and Jones[8] in England. There are also available numerous test results on T-beams, reported earlier by Bach and Graf.[18]

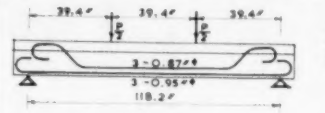
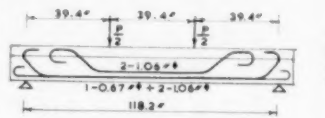
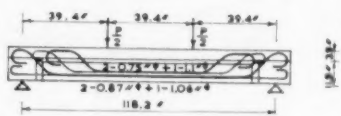
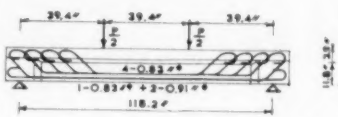
In these tests the arrangement of bent-up bars was based on the truss analogy, using single-, double-, and triple-lattice girders. The bent-up bars were inclined at 45 deg to the axis of the beam, and represented the tension web members and the concrete providing the compression web members meeting the bent-up bars at the level of the center of the compression zone. Table 14 summarizes the results of these tests, and Fig. 21 shows the crack patterns for Beams No. 29, 31, and 38 at failure; Beam No. 42 failed in a manner similar to Beam No. 38.

In the single lattice type of beam with the first diagonal member (reckoned from the support) in compression (Beam No. 29) the initial diagonal tension crack developed from a flexure-tension crack at the section where the inclined bar was bent-up. This section is particularly vulnerable since, owing to the change of the area of the tension steel, there is a sudden change in strain in the concrete at the bottom of the beam. This behavior is somewhat similar to that of the beams with cutoff main steel (discussed in Part 2)* where a sudden change in strain in the main steel produces a potentially critical section. On increase in load, further diagonal tension cracks formed while the initial crack, not meeting any bent-up bars, widened and extended first up to the flange, and later along the tension steel toward the support with a splitting of the concrete at this level. When the splitting reached the support the beam end was destroyed by the action of the hooks; simultaneously, splitting between the flange and the web occurred, and the concrete in the flange at midspan was crushed.

In the beam with the first diagonal member (reckoned from the support) in tension (Beam No. 31) two diagonal tension cracks formed near the load point and pointing toward it. Both these cracks encountered bent-up bars, and therefore remained fine. When the load increased a further diagonal tension crack formed nearer the support, passing through the section where the inclined bar is bent-up. This

*Taub, J., and Neville, A. M., "Resistance to Shear of Reinforced Concrete Beams. Part 2—Beams with Vertical Stirrups," ACI JOURNAL, V. 32, No. 3, Sept. 1960 (*Proceedings* V. 57), pp. 315-336.

TABLE 14—SHEAR STRENGTH OF T-BEAMS WITH DIFFERENT BENT-UP BAR ARRANGEMENTS

Investigator	Beam No.	BEAM DETAILS	$\frac{a}{d}$	f'_{cu} (+) psi	p (++) percent	P_u kips	v_u psi	$\frac{v_u}{f_{cu}}$	Calculated stress at midspan f_c psi	Calculated stress at midspan f_s psi	Calculated stress in bent-up bars (xx) f_w – psi	Mode of failure
Bach and Graf (16)	29		2.9	3440	3.6	92.6	500	0.145	2680	39,500	37,000	Destruction of beam end by hooks of straight bars, crushing of concrete in the flange at mid-span.
	31		2.9	3440	3.6	83.8	450	0.131	2500	36,000	33,300	Destruction of flange by hooks of bent-up bars.
	38		2.9	3440	3.6	109.1	590	0.171	3160	45,800	43,800	Yield of steel at midspan.
	42		2.9	3440	3.6	104.3	564	0.164	3060	43,000	45,700	Yield of steel at midspan.

Notes: (+) Measured on 7.9 in. cubes.
(++) Based on width of stem
(x) The cross-sectional areas of bent up bars were such that the stress in them, calculated by the truss analogy method, was approximately equal to the stress in the tension steel at midspan.
All steel consists of plain round bars.

crack did not meet any bent-up bars in its upward path, and extended, therefore, the full height of the web. Under further increase in load this crack widened considerably, and continued as a split between the web and the flange. Failure occurred by the destruction of the beam owing to the hooks of the bent-up bars causing splitting of the flange.

The double-lattice type of beam (No. 38) developed the first diagonal tension crack from a flexure-tension crack at the section where a bar was bent-up nearest to midspan. With increase in load, further diagonal tension cracks developed nearer to the support. All the diagonal tension cracks encountered one or more inclined bars and, as a consequence, never developed into more than fine cracks; no splitting at the level of main steel and no destruction of the beam end took place, and the beam ultimately failed in flexure at midspan.

The triple-lattice type of beam (No. 42) failed in a manner similar to Beam No. 38. The lower failing load can be explained by the lower yield point of the steel used in this beam as compared with the steel used in the previous three beams.

From these descriptions of failures, as well as the crack-pattern diagrams of Fig. 21, it can be seen that the double- and triple-lattice type beams exhibited adequate resistance to shear and reached their full load-carrying capacity in flexure. The beams modeled on the single-lattice failed in shear-tension (by splitting of the concrete and the destruction of the beam ends) in a manner similar to beams with no web reinforcement. Nevertheless, the bent-up bars contributed considerably to the shear resistance so that the strength in shear of beams with single-lattice type of bent-up bars was about 60 percent higher than that of beams with no web reinforcement.

INFLUENCE OF THE SPACING OF BENT-UP BARS

The question arises, therefore, as to what arrangement of bent-up bars ensures full protection against failure in shear. It may be recalled that the original concept of bent-up bars is based on the assumption that they lie in the direction of the principal tensile stress in concrete, and it has been observed that in beams with full shear reinforcement, the crack pattern corresponds approximately to the stress trajectories.

Fig. 22(a) shows a section along such a 45 deg line up to the neutral axis of the beam, and then continuing vertically through the compression zone to the top surface. Once the diagonal tension crack has opened over a section such as a-b, the inclined bar, together with the longitudinal reinforcement, must carry the resultant T_d of the principal tensile stress v over the section a-b times the width of the web. For convenience, the bent-up bar in Fig. 22(a) is shown to coincide with the resultant T_d. It is clear, however, that, since the position of a crack in the beam cannot be a priori determined, the bent-up bars

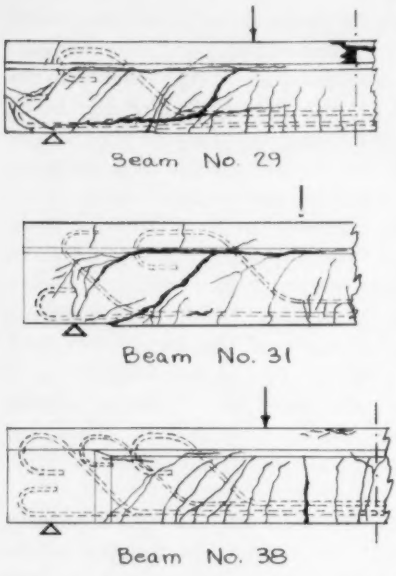


Fig. 21—Crack patterns of Beams No. 29, 31, and 38 (Table 14)

have to be spaced in such a manner that whatever the position of a crack it will encounter at least one bent-up bar.

From an examination of the crack patterns in Fig. 21 it can be seen that the diagonal tension cracks form at sections where a bar is bent-up. These sections represent, therefore, potential danger, as far as shear failure is concerned, and an investigation of the distribution of forces at such a section [Fig. 22(b)] should be worthwhile. T_d is the resultant of the principal tension stresses, and once the diagonal tension crack has formed, T_d has to be carried by the bent-up bars. The bar at a can carry a part of this force, and for equilibrium another bent-up bar is required to take the remainder of the force T_d. Such a bar cannot be farther away than b.

Thus the largest spacing of bent-up bars to ensure full resistance to shear is $2d(1-k)$, measured parallel with the axis of the beam. When this limitation of spacing is observed the equilibrium shown in Fig. 22(b) is maintained and only fine cracks open: the flexural load capacity of the beam is unimpaired.

If, however, the distance between bent-up bars exceeds $2d(1-k)$ the diagonal tension crack can extend and widen, and it will lead to a premature failure of the beam, no matter how big the cross-sectional area of the bent-up bars.

It can be seen that the distance of $2d(1-k)$ is smaller than the distance between bent-up bars in a single-lattice arrangement ($2jd$) but greater than in a double-lattice arrangement (jd). This agrees with the conclusion from Table 14 where a beam with bent-up bars spaced at jd was shown to be fully protected against failure in shear.

It now remains to prove experimentally that a beam with web reinforcement consisting of bent-up bars spaced at $2d(1-k)$, as proposed, is also so protected. Among the beams tested by Bach and Graf[18] there can be found one with such a spacing. This beam, No. 47, shown in Fig. 23, behaved in a manner similar to a beam of the double-lattice type: only fine cracks developed, and failure took place in flexure under a load approximately equal to the ultimate load of the beam

modeled on a double-lattice girder. For comparison, it may be recalled that single-lattice type beams failed in the shear, and it is believed that this was so because the spacing of bent-up bars exceeded $2d\ (1 - k)$.

According to the lattice theory the double- and the single-lattice beams should have been equally strong and, provided the cross-sectional area of the bent-up bars was adequate, neither should have failed in shear. Thus the actual behavior is substantially at variance with the lattice theory. Furthermore, there are serious theoretical objections to the lattice theory. In particular, the girder members assumed to exist in a reinforced concrete beam with bent-up bars are not pin-jointed as in a true truss; nor are the loads applied at the joints, and consequently the forces in the assumed members are not axial.

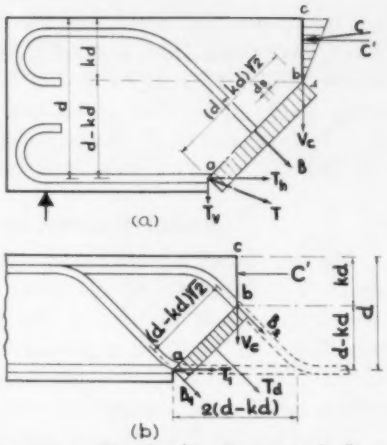


Fig. 22 — Free-body diagram before cracking. (a) General section. (b) Section where a bar is bent up

A decrease in the spacing of bent-up bars or inclined stirrups below the maximum of $2d(1 - k)$ affects favorably the load capacity of a beam, but the effects of a change in spacing are much smaller. For instance, Jones'[8] tests on rectangular beams subjected to a symmetrical two-point loading, with the a/d ratio of 4.07, show the following average values of ultimate loads for 11 beams of concrete strength between 1600 and 11,000 psi:

Spacing of inclined stirrups measured along axis of beam	*Ultimate load*
approximately jd	46 kips
$d(1 - k)$	55 kips

All these beams seem to have failed in shear-tension.

Wilby's[17] test data show that the load-carrying capacity of beams increases as the spacing of stirrups inclined 45 deg decreases, for a constant percentage of web reinforcement. The stirrups do not become entirely ineffective at spacings exceeding $2d$.

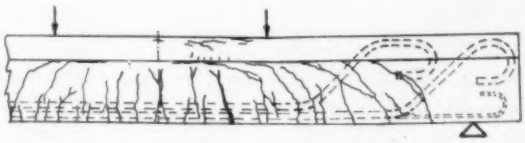


Fig. 23 — Crack pattern of Beam No. 47

The above discussion on the spacing of bent-up bars is believed to have suggested that a condition necessary for a full protection from shear failure of a beam is that the spacing of bent-up bars be not more than $2d(1-k)$. This condition is, however, not sufficient as a beam can fail in shear if the cross-sectional area of the bent-up bars is inadequate. Such a case will now be considered.

REDISTRIBUTION OF INTERNAL FORCES

Fig. 24 shows a portion of a beam bounded by the diagonal tension crack that has reached the level of main steel, and by a vertical section above the crack up to the top surface of the beam. It is immaterial whether the inclined crack developed from a flexure crack or first formed at middepth of the beam. The deformation of the beam is represented diagrammatically by Fig. 25.

Referring to Fig. 24 and equating horizontal forces, we have:

$$T_{h2} = C' - B_h$$

also

$$C' = T_{h1}$$

hence

$$T_{h2} = T_{h1} - B_h \qquad (5)$$

Considering the vertical forces, we have:

$$T_{v2} = V - V_c - B_v \qquad (6)$$

Under some load on the beam the stress in the bent-up bars reaches the yield point, and they now enter the plastic range. On further increase in load the diagonal tension crack opens wider with a consequent increase in strain in the bent-up bars crossed by the crack but the force in them remains unchanged. Thus their role as shear reinforcement has come to an end. *From now on* a reinforced concrete beam with shear reinforcement consisting of bent-up bars only, behaves *like a beam without web reinforcement* in which the diagonal tension crack has developed, and a redistribution of internal forces takes place.

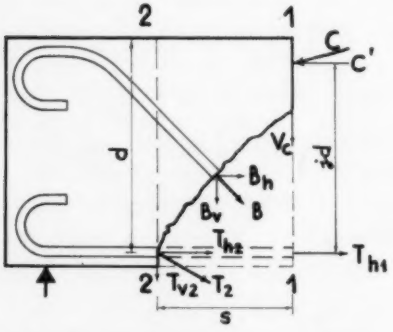


Fig. 24—Free-body diagram after the diagonal tension crack has reached the level of the tension steel

Since the magnitude of B_h cannot increase further when the load on the beam increases it follows that the increase in tension T_{h2} is equal to the increase in T_{h1}. Thus the force T_{h2} now becomes considerably larger than would be calculated for Section 2-2 by the elastic theory.

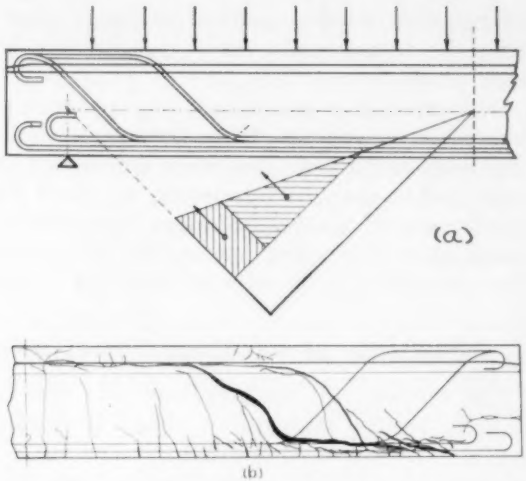


Fig. 25 — Beam No. 1032. (a) Design of shear reinforcement. (b) Crack pattern

Likewise, it is apparent that, once the inclined bars have yielded, the magnitude of the vertical component of the force in the inclined bar, B_v, cannot increase further. Thus any increase in the applied shearing force will be taken by the concrete above the inclined crack and also by the main reinforcement. The shearing force in the main steel, coupled with direction tension, can result in progressive splitting of the beam along this steel from the diagonal tension crack back to the support, in the same manner as in beams without web reinforcement.

It is important to note that while in the case of flexure failure of a beam the yield of main steel results in the collapse of the beam, the shear failure does not occur immediately when the bent-up bars have yielded.

Although no data on beams with bent-up bars are available, an experimental confirmation of the above statement can be inferred from some of Moody, Viest, Elstner, and Hognestad's[3] tests on restrained beams in which strains in the inclined stirrups were measured. In the data given for three of these beams (No. 13b, 14b, and 15b) the yield of stirrups occurred under loads of 134, 178, and 241 kips, respectively, whereas the corresponding ultimate loads were 170, 240, and 305 kips. All three beams had the same main reinforcement but varying amounts of web reinforcement, viz., r of 0.74, 1.35, and 2.09 percent. The percentage increase in load which the beams could carry, expressed as a fraction of the load producing yield of the inclined bars, was of the same order in all three beams.

Similar behavior can be observed in Wilby's[17] tests on simply supported beams with inclined stirrups, and also in Moretto's[31] tests on

beams with welded inclined stirrups: after the yield of the inclined bars the beams could carry considerably higher loads before failure took place.

The equilibrium of forces acting on a section of the beam with bent-up bars after the process of redistribution of internal forces has commenced will now be studied. This stage corresponds to the yield of the bent-up bars, and it may be interesting to recall that in beams without web reinforcement the redistribution starts much earlier, namely when the diagonal tension crack reaches the main reinforcement. In either case this redistribution progresses until the final collapse of the beam.

INFLUENCE OF CROSS-SECTIONAL AREA OF BENT-UP BARS

The influence of the cross-sectional area of bent-up bars at a given spacing on the strength in shear of reinforced concrete beams was investigated by Bach and Graf.[36] The authors do not agree with some of Bach and Graf's interpretations, but by studying selected beams valuable conclusions can be drawn. Details of three of their T-beams subjected to a uniformly distributed load are given in Table 15. The layout of the bent-up bars was similar in all beams. The cross-sectional area of the bent-up bars was chosen in the following manner: in Beam No. 1026 the bent-up bars were assumed to take the whole shearing force (as prescribed by the British and German codes); in Beam No. 1025 the cross-sectional area of the bent-up bars was 50 percent of that calculated for Beam No. 1026, while the corresponding figure for Beam No. 1031 was 37 percent.

The cross-sectional area of the main reinforcement at midspan was the same in all beams but, since varying amounts of steel were bent-up in different beams, the amount of the main tension steel continuing up to the support varied. Consequently, in beams with bent-up bars of small cross-sectional area the reinforcement continuing up to the support was far in excess of that required to resist the applied bending moment.

From the crack pattern of the beam with full shear reinforcement (No. 1026, shown in Table 15) it can be seen that numerous diagonal tension cracks opened in the outer parts of the beam right up to the supports but all these cracks remained extremely fine throughout, and did not impair the composite action of the beam. Flexure-tension cracks opened at midspan and led to the failure of the beam through the yield of the steel.

The crack pattern of Beam No. 1025, which had "50 percent shear reinforcement" is shown in Table 15. Flexure- tension cracks opened in the middle portion of the beam, and numerous diagonal tension cracks in the outer parts. Under increasing load some of the inclined cracks opened wider and the further behavior of the beam appeared to

be typical of a beam with inadequate web reinforcement: the diagonal tension crack opened widely, the main steel was pressed down, a horizontal crack along it opened toward the support, the hooks of the tension steel caused cracking at the beam ends, and a vertical crack was formed between the web and the flange. At the same time, however, one of the flexure cracks increased in size, and it was that crack which eventually led to the collapse of the beam through the yield of the main tension steel at midspan. The failing load of this beam (264 kips) was equal to the load of the beam with "full shear reinforcement" (262 kips).

This high strength of Beam No. 1025 is of interest since its crack pattern was dissimilar from the beam with "full shear reinforcement." It is clear that the yield of the bent-up bars did not produce failure but led to a redistribution of internal forces in the outer parts of the beam. As a result, the actual forces differed considerably from those calculated by the elastic theory: the stress in the bent-up bars never reached the calculated value of 104,000 psi but, on the other hand, the straight bars between the diagonal tension crack and the near support were stressed much higher than would be expected from the elastic theory. It may be remembered that the smaller the proportion of

TABLE 15 — INFLUENCE OF CROSS-SECTIONAL AREA OF BENT-UP BARS ON SHEAR STRENGTH OF T-BEAMS

Investigator	Beam No.	BEAM DETAILS	f'_{cu} (a) psi	p percent	r as a percentage of required design reinforcement	P_u kips	v_u psi	$\frac{v_u}{f'_{cu}}$	Calculated stress in bent-up bars (x) f_w = psi	Mode of failure
Bach and Graf (26)	1026		4000	2.4	100	262	595	0.149	53,000	Yield of steel at midspan
	1025		4000	2.4	50	265	602	0.150	104,000	Yield of steel at midspan
	1031		4000	2.4	37	212	482	0.123	119,000	Shear-tension
	1024		4000	2.4	0	108	246	0.061	—	Shear-tension

Notes: (a) Measured on 7.9 in. cubes.
(x) Yield point of reinforcement is 38,000 – 52,000 psi depending on size.
All steel consists of plain round bars.

TABLE 16—CALCULATED STRESSES IN WELDED INCLINED STIRRUPS AT FAILURE

Investigator	Beam No.	BEAM DETAILS	f'_c psi	p percent	r percent	V_u kips	V_p kips	Calculated inclined stirrup stress on the basis of V_u(+) psi	Calculated inclined stirrup stress on the basis of $(V_u + V_p)$ (++) psi	Mode of failure
Moretto [31]	2-D¼		3455	4.00	0.28	70.3	7.9	292,000	259,000	Shear
	2-D⅜	As beam 2-D¼, but stirrups ⅜" ϕ	3685	4.00	0.615	71.8	7.9	133,000	118,000	Shear

Notes: (+) British code design method
(++) ACI Standard design method

reinforcement bent-up the larger was the area of the straight bars in the outer parts of the beam, and for this reason these straight bars in Beam No. 1025 could carry an increasing load until the failure of the beam by the yield of the main steel at midspan.

The third beam chosen for comparison (No. 1031) had a "37 percent shear reinforcement," as defined earlier, and failed in shear-tension under a load of 212 kips. This comparatively high load is due to the fact that in the outer parts of the beam there was a large cross-sectional area of straight bars, whose action has been described above.

It may be interesting to note that a beam with the same cross-sectional area of reinforcement at midspan as the three beams described above, but with all the main steel carried straight to the supports, i.e., without web reinforcement (Beam No. 1024), failed in shear-tension under a load of only 108 kips. This is barely half the ultimate load of a beam with only "37 percent shear reinforcement." The tremendous importance of the provision of web reinforcement is self-evident.

Similar effects are shown by the test results of a more recent investigation by Moretto[31] on rectangular beams with welded inclined stirrups. For instance, Beams No. 2-D¼ and 2-D⅜, described in Table 16, had practically the same ultimate load although the former had only half the area of the web reinforcement of the latter. Beam No.

2-D½ failed in flexure under a higher ultimate load. The values of the nominal shearing stress v corresponding to the yield of inclined stirrups are of interest: for Beam No. 2-D¼ $v = 534$ psi, and for Beam No. 2-D⅜ $v = 638$ psi. Thus for a 100 percent increase in stirrup size the increase in the nominal shearing stress at the yield of inclined stirrups is barely 20 percent. It is not surprising, therefore, that the stirrup stresses, calculated according to the current theory and given in Table 16, are even more striking than the stresses in the bent-up bars of Bach and Graf's,[36] quoted in Table 15.

It may be recalled that the behavior of Beams No. III-30 and III-31 with vertical stirrups, tested by Moody, Viest, Elstner, and Hognestad[3] and discussed in Part 2 of this series (Fig. 20), is similar to that of Moretto's beams.

CONTRIBUTION OF BEAM ELEMENTS TO SHEAR RESISTANCE

An interesting attempt to directly measure the separate contributions to the shear resistance of a beam made by the concrete above the diagonal tension crack, by the stirrups inclined 45 deg, and by the main steel, was made by Jones.[8] He tested three rectangular beams with a preformed gap on the line of the expected diagonal tension crack, using a special measuring device in place of the compression zone above the crack. The beams had an area of tension reinforcement of 9 percent, and compression reinforcement was also provided. All the beams were subjected to symmetrical two-point loading with the a/d ratio of 4.07.

Under low loads the inclined stirrups carried about 60 percent of the applied shearing force, the remainder being resisted by the compression zone of the beam and by the compression and tension reinforcement of square twisted steel.

When the load on the beam reached 63 percent of its ultimate load the inclined stirrups spanning the preformed gap yielded. This was shown by a slight drop in the stirrup forces under a constant load on the beam, and by a corresponding increase in the shearing force in the tension steel. This then was the commencement of the redistribution of internal forces. Further increase in the applied shearing force was shared by the compression zone and by the shearing force in the main steel.

Under a load equal to 90 percent of the ultimate the inclined stirrups resisted 33 percent of the shearing force, the tension steel 30 percent, and the compression zone the remaining 37 percent. It should be remembered that the crack was preformed.

Care is necessary in the interpretation of these results since they refer, in particular, to one value of the a/d ratio only. Nevertheless, these measurements afford an interesting and convincing proof of the redistribution of internal forces and they also emphasize the fact that

web reinforcement is only one of the elements contributing to the shear strength of a beam.

An indirect proof of this statement can be obtained from a calculation of stresses in inclined stirrups by the elastic theory. As an example, the calculated results for some of Moretto's[31] beams are given in Table 16: the values calculated are clearly impossible.

The effects of compression reinforcement per se have not been definitely established. Laupa, Siess, and Newmark[20] believe that this reinforcement contributes to the shear strength of a beam. Wilby[17] found a similar effect for small amounts of compression reinforcement but an opposite effect for large amounts of compression steel. For beams with vertical stirrups, Larsson[29] found compression reinforcement to have no effect on the shear strength of a beam, and it would be reasonable to expect this effect to be similar in beams with stirrups and in beams with bent-up bars.

There does not seem to be sufficient evidence available on the influence of the various factors, such as the a/d ratio, the compressive strength of concrete, etc., on the shear strength of beams with bent-up bars or stirrups inclined 45 deg. Pending such information being available, it seems reasonable to assume that the effects of the various factors are similar to the effects on beams with vertical stirrups, described in Part 2 of this series. There is a clear need, however, for experimental data to be obtained.

BEHAVIOR OF BEAMS SUBJECTED TO UNIFORMLY DISTRIBUTED AND NONSYMMETRICAL LOADINGS

It may be remembered that Beams No. 1026, 1025, and 1031 were subjected to a uniformly distributed load. A further insight into the action of shear in reinforced concrete beams can be gained by considering a beam with bent-up bars provided in the proximity of the supports only, i.e., in the zone which, according to the elastic theory, is subjected to a maximum shearing stress. The design of this T-beam (No. 1032), subjected to a uniformly distributed load,[36] was similar to the American practice, viz., a shearing stress of 0.021 f_c' was assumed to be carried by the concrete, and the bent-up bars were provided to take the excess shearing stress which totaled 0.055 f_c', as shown in Fig. 5(a).

Fig. 5(b) shows the crack pattern of this beam: first, flexure tension cracks formed in the center part of the beam, followed by diagonal tension cracks in the outer parts of the beam. Each of these inclined cracks encountered the bent-up bars. When the load on the beam increased further, a crack opened at a section where the inner inclined bar is bent-up. Since this crack did not encounter any bent-up bars it opened wide and led to the pressing down of the main steel at the

bottom end of the crack, followed by splitting of the concrete at the level of the tension steel, and destruction of the beam end by the hooks of this steel. The upper end of the critical crack extended along the junction of the flange with the fillet of the web-flange connection up to the midspan, where crushing of the concrete in the outer part of the flange took place. This led to the collapse of the beam under a load of 203 kips, which is 77 percent of the failing load of the beam with "full shear reinforcement," (Beam No. 1026).

The failure of Beam No. 1032 should draw attention to the fact that in beams subjected to a uniformly distributed load, shear failure can occur at a considerable distance from the supports — in the part of the beam not subjected to the highest shearing force. This has to be borne in mind in the design of web reinforcement.

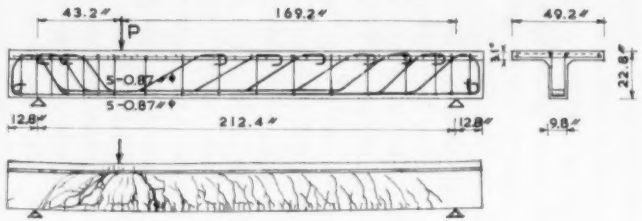
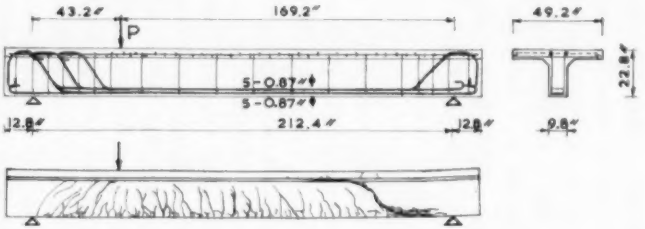
In Part 2, the behavior of beams with vertical stirrups under the action of nonsymmetrical loading was discussed. Comparatively few tests under such loading are available for beams with bent-up bars, but some tests of Graf's[37] confirm the authors' recommendation that web reinforcement should generally be provided throughout the beam and not only in the zone of the maximum shear.

T-beams No. 1203-1206 were tested under one-point load such that the values of the a/d ratio for the two shear spans were 2.09 and 8.18. Those beams which had bent-up bars in the smaller shear span only (with some vertical stirrups throughout) failed under an average load of 127 kips, their collapse being due to a sudden shear-tension failure in the larger shear span (Table 17). On the other hand the beams which were reinforced with bent-up bars along their entire length failed in flexure-tension under an average load of 165 kips, thus showing an increase of 30 percent over the previous series.

Some further tests were made on T-beams subjected to three equal point loads at midspan and sixth points nearest to supports. Those beams which had bent-up bars along their entire length failed under an average load of 213-5 kips (Table 18); those with bent-up bars between the support and the near load only, i.e., in the outer sixths, (with some vertical stirrups throughout) failed under a load of 203.5 kips when all the tension reinforcement was carried through without cutoff (Table 17); and under 179.5 kips when some tension reinforcement was cut off in the tension zone, when no longer required from bending moment considerations. This influence of the cutting-off of tension reinforcement in the tension zone of the beam was discussed in Part 1.

These test results, together with those on beams subjected to a uniformly distributed load, as well as the test data of Parts 1 and 2, indicate that failure does not necessarily occur in the zone of maximum shear, and emphasize the importance of the provision of web reinforce-

TABLE 17 — STRENGTH OF T-BEAMS UNDER A NONSYMMETRICAL POINT LOAD

Investigator	Beam No.	BEAM DETAILS	$f_{cu}^{(+)}$ psi	P_u kips	v_u psi	$\frac{v_u}{f_{cu}}$	Mode of failure
Bach and Graf [37]	1203 1205		1810	165	932	0.515	Flexure-tension.
	1204 1206		1800	127	714	0.396	Shear

Notes: (+) Measured on 7.9 in. cubes
All steel consists of plain round bars

ment throughout the length of a beam. These design aspects will be discussed in more detail in connection with the use of bent-up bars and vertical stirrups combined, in Part 4.

BEAMS WITH INADEQUATE WEB REINFORCEMENT CONSISTING OF BENT-UP BARS

It would be of interest to compare now the two basic types of inadequate web reinforcement consisting of bent-up bars only, viz., when they are spaced too far apart, and when their spacing is correct but the cross-sectional area is too small.

Typical examples of too large spacing were Beams No. 29 and 31 of Table 14. In both these beams the critical diagonal tension crack passed through the point where the inclined bar was bent-up, and this crack did not encounter any other bent-up bars. Thus the distribution of internal forces here is similar to that in a beam without web reinforcement, and the same equations of equilibrium are, therefore, applicable:

$$\left.\begin{aligned} T_v &= V - V_c \\ T_h &= C' \end{aligned}\right\} \quad \text{and} \quad \ldots\ldots (7)$$

Fig. 26 shows the case when the crack passing through the critical section encounters another inclined bar, i.e., when the spacing of the bent-up bars is adequate. In this case, we have:

$$\left.\begin{aligned} T_v &= V - V_c - B_v \\ T_h &= C' - B_h \end{aligned}\right\} \quad \text{and} \quad \ldots\ldots (8)$$

Comparing Eq. (7) and (8) it can be seen that the forces T_h and T_v, i.e., those forces which in the case of shear-tension failure are directly responsible for the collapse of the beam, are considerably higher in case (7) than in case (8). Thus, when the spacing of the bent-up bars is too large the load capacity of the beam is considerably reduced *however large* their cross-sectional area. On the other hand, when the spacing of the bent-up bars is satisfactory they contribute materially to the shear strength of the beam even if their cross-sectional area is inadequate. This shows the paramount importance in design of the correct spacing of bent-up bars. Although the tests described indicate the order of the size of bent-up bars required, it is not possible, on the basis of the information available, to calculate this size with any precision: the bent-up bars are only one of the components of the

*Taub, J., and Neville, A. M., "Resistance to Shear of Reinforced Concrete Beams. Part 1—Beams without Web Reinforcement," ACI JOURNAL, V. 32, No. 2, Aug. 1960 (*Proceedings* V. 57), pp. 193-220.

After this paper had been written an interesting confirmation was obtained of shear failure taking place in a zone where the shearing force is less than a maximum: Rodriguez, José J., Bianchini, Albert C., Viest, Ivan M., and Kesler, Clyde E., "Shear Strength of Two-Span Continuous Reinforced Concrete Beams," ACI JOURNAL, V. 30, No. 10, Apr. 1959 (*Proceedings* V. 55) pp. 1089-1130, found that "in five beams in Series N and Series A a diagonal tension crack formed in Shear Span 2, where in accordance with ACI 318-51, web reinforcement was not required."

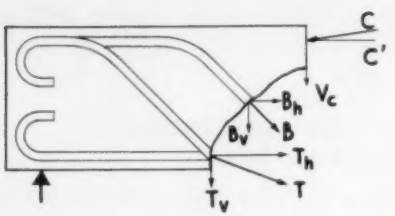
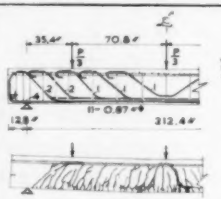
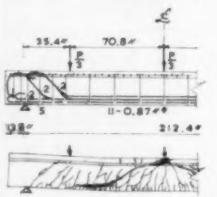
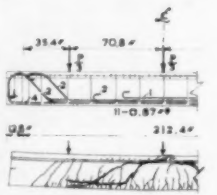


Fig. 26 — Free-body diagram for the case when the spacing of bent-up bars is adequate

shear reinforcement of a beam. It will be shown in Part 4 that although full shear protection is possible using bent-up bars alone, the most effective web reinforcement consists of a combination of bent-up bars (of nominal size) and vertical stirrups.

It may be recalled that Beams No. 29 and 31 of Table 14, differing in the arrangement of the bent-up bars, exhibited different behavior at failure. In Beam No. 29 the support-to-support bars of the tension steel were larger than the bent-up bars, and that is why the former carried a larger force. Consequently, their hooks were more destructive and split the beam end. On the other hand, in Beam No. 31 there was

TABLE 18 — STRENGTH OF T-BEAMS UNDER THREE-POINT LOADING

Investigator	Beam No.	BEAM DETAILS	f_{cu} (+) psi	P_u kips	v_u psi	$\frac{v_u}{f_{cu}}$	Mode of failure
Bach and Graf (37)	1197 1200		1990	213.5	1196	0.600	Flexure-tension
	1198 1201		1990	203.5	1140	0.572	Tension-shear at midspan
	1199 1202		1990	179.5	1005	0.505	Shear at midspan

Notes: (+) Measured on 7.9 in. cubes
All steel consists of plain round bars

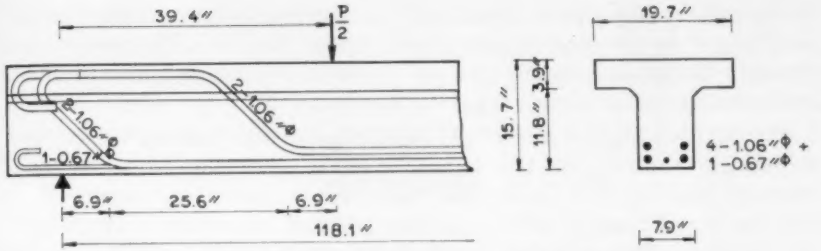
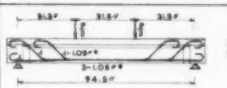
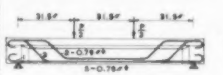
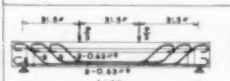
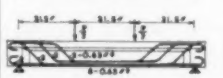


Fig. 27—Anchorage of bent-up bars of Beam No. 33

only one small bar straight between the supports. The force in the large bent-up bars was considerable and that is why *their* hooks caused the collapse of the beam.

Referring again to Beam No. 31 (Table 14) it may be noted that the inner bent-up bars were anchored immediately as they entered the compression zone. Let us now consider another beam, No. 33 (Fig. 27), differing from Beam No. 31 only in that the inner bent-up bars on reaching the compression zone continued horizontally and ended in hooks over the supports. Beam No. 33 failed by the destruction of the

TABLE 19 — INFLUENCE OF ANCHORAGE OF BENT-UP BARS

Investigator	Beam No.	BEAM DETAILS	$\frac{a}{d}$	f_{cu} psi	$p^{(a)}$ percent	P_u kips	v_u psi	$\frac{v_u}{f_{cu}}$	Increase in $\frac{v_u}{f_{cu}}$ due to anchorage of bent-up bars in the beam ends – percent	Mode of failure
Seliger	3		2.9	3560	3.6	37.4	306	0.086		Destruction of beam end by hooks
	7		2.9	3560	3.6	56.0	457	0.129	50	Destruction of beam end by hooks
	9		2.9	3560	3.5	58.2	500	0.141		Destruction of beam end by hooks
	13		2.9	3560	3.5	64.0	549	0.155	10	Destruction of beam end by hooks
	15		2.9	3560	3.6	67.0	573	0.161		Destruction of beam end by hooks
	19		2.9	3560	3.6	71.8	617	0.174	7	Yield of steel at midspan

Notes: (a) Based on width of stem.
All steel consists of plain round bars.

beam end by the upper hooks, i.e., in the same manner as, but at a load some 10 percent higher than, Beam No. 31. This increase in strength is believed to be due to the better anchorage of the hooks in Beam No. 33 as compared with Beam No. 31.

The relative importance of the anchorage of the bent-up bars appears to depend, however, on the degree of shear protection afforded by the bent-up bars in the particular case: the better this protection the smaller the influence of the continuation of the bent-up bars up to the support. This is clearly borne out by Saliger's[38] tests, summarized in Table 19.

The full problem of anchorage of reinforcement will be discussed in Part 5 and the use of bent-up bars and stirrups inclined 45 deg will be compared in Part 4.

CONCLUSIONS

The yield of bent-up bars or inclined stirrups has been shown to lead to a redistribution of internal forces. The subsequent contribution of the beam elements to its shear strength governs this strength; hence there is a lack of proportionality between the area of web reinforcement and the shear strength of a beam.

It has been shown that bent-up bars alone can provide full protection from shear failure of a reinforced concrete beam. The conditions necessary for such complete protection are that the spacing of the bent-up bars be not more than $2d(1 - k)$, and that their cross-sectional area be sufficient to carry the principal tension across the diagonal tension crack. The inclined bars carry only a part of the shearing force on the beam section, the remainder being resisted by the concrete above the diagonal tension crack and by the shear in the main tension reinforcement.

If the bent-up bars are either spaced too far apart or their cross-sectional area is inadequate the shear resistance of the beam is lowered, and it will fail in shear under a load lower than that which would cause failure in flexure. The influence of inadequate spacing of the bent-up bars is considerably greater than that of insufficient area, and will result in a much lower ultimate load.

Tests on beams subjected to a uniformly distributed load and to various systems of point loading have shown that shear failure can occur in the part of the beam subjected to a shearing force less than the maximum on the beam. For this reason, if under ultimate flexure load the stress at such sections exceeds the cracking stress, the web reinforcement should be provided throughout the length of the beam; the design of web reinforcement of beams subjected to a uniformly distributed load requires particular care.

Although the general principles of the behavior of reinforced concrete beams under the action of shear have been established and suggestions on the spacing of bent-up bars have been made, no specific design formulas are given; further tests to establish quantitatively the various factors influencing the shear strength of a beam are yet to be made.

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36. Bach, C., and Graf, O., "Versuche mit Eingespannten Eisenbetonbalken," *Bulletin* No. 48, Deutscher Ausschuss für Eisenbeton, 1921 (published by W. Ernst & Sohn, Berlin).

37. Graf, O., "Versuche mit Eisenbetonbalken zur Ermittlung der Widerstandafähigkeit verschiedener Bewehrung gegen Schuckräfte. Sechster Teil," *Bulletin* No. 67, Deutscher Ausschuss für Eisenbeton, 1931 (published by W. Ernst & Sohn, Berlin).

38. Saliger, R., *Der Stahlbetonbau*, 7th Edition, Franz Deuticke, Vienna, 1949.

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*References are numbered consecutively beginning with Part 1 of this five-part series.

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CURRENT REVIEWS

of Significant Contributions in Foreign and Domestic Publications

Bridges

Exceptional job engineering pays off on bridge construction

IRA E. WILLIAMS, *Civil Engineering*, V. 30, No. 1, Jan. 1960, pp. 60-62

The construction of a highway bridge over Lake Tecoma has evolved three rather unique expediting techniques. (1) The requirement that concrete for the piles be placed in the dry led to the development of a self-stripping watertight form some 70 ft high. (2) To insure accurate placement of piles, a 40 ft long, 30 ton pile template was constructed. This huge template contains an accurate guide for each foundation pile. (3) Superstructure erection consisted of placing four 130-ft girders side by side (9 ft 4 in. on centers) and was accomplished by assembling them on shore by putting the girders on a steel falsework built on barges and then floating each one into position.

Vacuum concrete in bridge construction (Vacuum-behandelter Beton im Brueckenbau)

GUNTHER BRUX, *Beton-Herstellung und Verwendung* (Düsseldorf), V. 9, No. 8, Aug. 1959, pp. 285-290

Reviewed by FERDINAND S. ROSTASY

Vacuum concrete, which is now widely used in construction practice, brings in bridge construction several important advantages: savings of time and cost due to accelerated strength development which permits an earlier transfer of prestressing forces, earlier use of the structure, faster reuse of formwork, etc. Vacuum treatment reduces the effective water-cement ratio thus removing excess water necessary for better workability. The result is a higher strength and wear resistance. Shrinkage is cut down to 50 percent and creep is reduced because the vacuum creates a more rigid structure of the aggregate particles. In several examples of French and German highway bridges the authors shows how vacuum concrete can yield economical and technically sound structures.

Construction

Design and construction of Navy's largest drydock

S. P. ZOLA and P. M. BOOTHE, *Proceedings*, ASCE, V. 86, WW 1, Mar. 1960, pp. 53-84

A technical description of the design and construction of the Navy's largest drydock at the Puget Sound Naval Shipyard, Bremerton, Wash. The relieved type dock will be 1180 ft long, 180 ft wide, and 61 ft deep. Both mass concrete and precast concrete units will be utilized.

Bus station in the Place Rogier, Brussels (La gare d'autobus de la Place Rogier, a Bruxelles)

J. MAUQUOY, *Precontrainte-Prestressing* (Brussels), V. 10, No. 15, 1959-60, pp. 19-21

Three types of members were used in the construction of this bus station. Columns, spaced 27.4 x 52 ft, were reinforced concrete; main beams were composites made of steel beams with lower flanges encased in precast con-

A part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 4, Oct. 1960, *Proceedings* V. 57. Address P.O. Box 4754, Redford Station, Detroit 19, Mich. Where the English title only is given in a review, the book or article reviewed is in English. If it is followed by a foreign title the work reviewed is in that language. In those cases where the foreign title cannot conveniently be set in type or is not available, the language of the original article is indicated in parentheses following the English title. Copies of articles or books reviewed are *not* available through ACI. Available addresses of publishers are listed in the June "Current Reviews" each year. In most cases ACI can furnish addresses of publications added later.

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crete on which were placed secondary beams of prestressed concrete. Finally, the floor slab, which also encased the upper part of the steel beams, was cast in place.

In this same issue a number of other structures utilizing this composite system of construction are described.

Buildings with many floors

Engineering (London), V. 188, No. 4884, Nov. 27, 1959, pp. 536-537

Reviewed by ARON L. MIRSKY

Necessity forced British designers a few years ago to switch from structural steel to reinforced concrete. This in turn led to better concrete, precasting, prestressing, and other developments. Article summarizes current trends in both steel framed and concrete-framed multistory structures.

Synchroton concreting calls for thousandths-of-an-inch accuracy

Engineering News-Record, V. 163, No. 22, Nov. 26, 1959, pp. 40-42

Describes the construction of the alternating gradient synchrotron, a 25 billion-electron-volt proton accelerator at Brookhaven National Laboratory, Upton, N.Y. This is one of the most precise construction tasks ever undertaken with tolerances running ± 0.005 in.

Barrel arches top big terminal

Railway Age, V. 148, No. 20, May 16, 1960, pp. 56, 58

Reviewed by ARON L. MIRSKY

New Union Pacific freighthouse in East Los Angeles is 1332 x 104 ft in plan, with a completely unobstructed interior. Roof consists of 30 reinforced concrete barrel arches of lightweight concrete 4 in. thick, spanning 104 ft plus a 13 ft overhang serving as a canopy over a loading dock. Unusual dimensions of building were dictated by available land, situated between a thoroughfare and the tracks. Total cost of facility, including a two-story office building, a one-story salvage building, and a one-story block office, all in reinforced concrete, is $3 million.

Heavy concrete mixed in transit

Engineering News-Record, V. 163, No. 22, Nov. 26, 1959, pp. 43, 45, 48, 50

Discusses the high-density concrete, mix proportions, method of mixing (transit-mix trucks), and dimension control used in the construction of synchrotron shielding blocks and wall. Ilmenite was used for heavy aggregate and resulted in an average weight of 264 lb per cu ft with the cost of the installed blocks running around $190 per cu yd.

Prestressed circular concrete tanks for storage of fuel oils (Forspaendte cirkulaer-cylindriske beton beholdere til lagring af braendselsolier)

C. OSTENFELD and E. KALHAUGE *Cement och Betong* (Malmo), V. 33, No. 1, Mar. 1958, pp. 9-16

Reviewed by BENGT F. FRIBERG

The paper shows examples of prestressed circular concrete tanks for fuel oils employing full-thickness wall construction with the prestressing cables in tubes and lapped and tensioned at diametrically opposite points. The installations and their dimensions, including those with domed roofs, are listed in the following table.

The last mentioned tank was insulated with 4 in. of rock wool covered with aluminum sheet. In all tanks the heavy fuel oil is heated to about 120 F at removal.

Construction Techniques

Activation of mortars

V. SARETOK, *RILEM Bulletin* (Paris), No. 5, New Series, Dec. 1959, pp. 49-58

AUTHOR'S SUMMARY

The "activation" of mortars is a method of mixing mortars that is used in Scandinavian countries. It requires special equipment (activators); horizontal cylindrical vessels with an independent axis equipped with dashers.

The advantages of this method are: better homogeneity of mix, increase in the workability of the mortar, use of truly lean mixes, and increase in density and better frost-resistance. The

disadvantages are: great power consumption, and relatively small apparatus.

After a brief bibliographical survey, there is a comparison of results obtained on mortars with different methods of testing consistency and water retention. In conclusion, the Swedish Mo-meter gives satisfaction for the practical measurement of consistency in mortars; the flow-test likewise, except for mortars with air-entraining agent. The method of the modified cube (Wuerpel) and the Piepenburg-Nass dragging test can likewise be used. It is recommended that these tests be supplemented by a simple water retention measurement. These tests demonstrate the effect of activation on mortars: (1) water retention is considerably increased, (2) plasticity is increased, and (3) the strength remains, in general, unmodified.

The use of activation is more justified for coating mortars than for placed mortars.

Additions can be made to activated mortars: calcium chloride affects only the strength, which is slightly higher than for the control pieces; the plasticization is without effect; mortars that are activated and containing air-entraining agents lose a part of their air in the activation. The duration of the activating process is of great importance as it relates to the qualities of the mortar: 3 to 5 min are normal.

Waterproofing is an art

GRAYSON GILL, *Concrete Construction*, V. 5, No. 7, July 1960, pp. 186-189

Discusses some of the minimum requirements for effective waterproofing under various conditions. Successful waterproofing requires good details and specifications, proper materials and methods, and skilled, conscientious craftsmen.

Waterproofing for building structures below grade can be classified generally as integral waterproofing, membrane waterproofing, and hydrolithic waterproofing; each is described.

Moving concrete on the job site

Concrete Construction, V. 5, No. 7, July 1960, pp. 190-195

There is a wide choice of methods and equipment available for the all-important job of moving concrete from truck mixer to forms. Here are some factors to weigh in choosing the best method for the job. Methods and equipment described briefly include: buggies, wheelbarrows, loaders, hoists, buckets, chutes, conveyors, and pumps.

Safe handling of electronics encapsulating materials

JOHN DELMONTE, Presented at 7th Annual Western Safety Congress, Apr. 12, 1960, Los Angeles

Discusses benefits which can accrue from new encapsulation procedures, including epoxy resin formulations, and includes *recommendations for proper and safe handling practices.* Mention is made of the selection of the proper encapsulating material. Harmful consequences of improper handling procedures are discussed, and finally measures are summarized which may be followed to ensure safe handling of resin encapsulants.

Epoxy adhesives—Uses of a material new to construction

R. W. GAUL, *Western Construction*, V. 35, No. 3, Mar. 1960, pp. 63, 67, 70, 74
HIGHWAY RESEARCH ABSTRACTS V. 30, No. 6, June 1960

There are several different types of epoxy adhesives available: (1) Liquid for bonding fresh concrete to old concrete in such cases as concrete toppings to bridge decks, dam surfaces, and floor slabs; to build up gutter grades in proper levels, to eliminate depressions, and also to repair spalled concrete on vertical surfaces. (2) Paste is used to fill cracks and holes on vertical and overhead concrete, to bond stone, masonry, slate, marble, and other materials to concrete and asphalt pavements. (3) Grout to meet Air Force and Corps of Engineers specifications covering anchoring of steel

dowels in runway slabs and to repair cracks and spalls in concrete surfaces and joints.

Applications include the bonding of continuously extruded concrete curbing to existing roadways. Curbing extruded in a continuous line and bonded permanently to the roadway with a liquid epoxy adhesive costs less than $1 per lin ft for labor and materials.

Slippery cement concrete pavements in such areas as toll-booth approaches, bridges, and intersections can be made skid resistant by broadcasting grit over a thin layer of the epoxy liquid. An adhesive-aggregate layer just 15 mils thick may cost on the average of $2 per sq yd applied and will skid-proof the area for several years.

Chuck holes, potholes, and deep cracks in both concrete and asphalt pavements have been repaired successfully with an epoxy resin adhesive possessing high strength and a certain degree of flexibility so that the repair material gives with the pavement. Clean, dry aggregate and sand are added to the liquid resin to produce a thick consistency mortar which can be troweled easily to desired levels. Heat applied to the mortar will cure it in less than 30 min.

This type of heavy-duty mortar was used to repair chuck holes in several streets in Los Altos, Calif., in 1958. Examination of the repaired areas in 1960 revealed that the mortar was still intact, without any signs of cracking or breaking.

Reducing concrete aging time by means of electric current (in Polish)

T. Latomski, *Przeglad Budowlany* (Warsaw), No. 4, 1959, pp. 193-195
Polish Technical Abstracts No. 1 (37) 1960

In the course of performing concrete work in winter, electric heating of monolithic reinforced concrete constructions was introduced on various building sites. The weight of some of the reinforced concrete elements heated reached 10 tons. Electric current of 50 v used for heating was supplied from Russian TB 20 transformers. Power consumption did not exceed 70 kwhr per cu m of structure. Owing to the small number of TB 20 transformers available, Polish ETb 250 welding transformers were also used. Fully satisfactory results were obtained. Trial heating in mass production of prefabricated elements was effected using TB 20 transformers; hollow floor slabs were heated. Results obtained proved fully adequate to solve the electrical heating problems, this method ensuring considerable savings as compared with steam heating.

Air-powered placer halves concrete crew

Construction Equipment, V. 21, No. 6, June 1960, pp. 48-50

Describes the placement of concrete, during construction of an underground garage, by using a 600-cfm compressor to force the concrete through a 6-in. diameter supply line made up of 10-ft sections. This pipe has swivel points which give flexibility to the otherwise rigid pipe. By using this procedure the contractor was able to place 25 cu yd of concrete in 1 hr in an area which had only the day before been a tedious and expensive job.

New building techniques

J. L. Robinson, *Engineering* (London), V. 188, No. 4884, Nov. 27, 1959, pp. 547-548
Reviewed by Aron L. Mirsky

Some of the new building techniques are — prefabrication, on-site mechanization of operations, slipforms, liftslabs, portable power tools, and a relative stranger in such company — work study (time and motion study).

Bypass problems in concrete construction

Contractors and Engineers, V. 57, No. 6, June 1960, pp. 97-100

Reviews two problem saving techniques employed during the construction of a new dam which will be used to supply water for Newark, N. J. The first technique is the cableway system

which is used to haul concrete by bucket, thereby eliminating the truck delivery system. The second technique is used to cool down aggregate to 35 F, this was accomplished by the jet-stream-vacuum process. This was done to maintain the job temperature specification of concrete which was 60 F.

Engineering in big buildings

Engineering (London), V. 188, No. 4884, Nov. 27, 1959, pp. 541-542
Reviewed by Aron L. Mirsky

Pictorial review* of some of the new materials and devices being used in multistory buildings of all types.

*This serves to introduce a review of "Engineering in Big Buildings," composed of the papers by Mathews, Eden, and Robinson.

Tower cranes change building designs

J. F. Eden, *Engineering* (London), V. 188, No. 4884, Nov. 27, 1959, pp. 545-546
Reviewed by Aron L. Mirsky

The tower crane has speeded up the construction of tall buildings, particularly those with reinforced concrete frames in which many repetitive operations take place, such as placing prefabricated or reused forms, placing reinforcing steel, and concreting.

Many other aspects of construction have become mechanized—thus, ready-mixed concrete, precast wall panels, and slipforms, all are making their imprint on designs.

Shore up your profits

Construction Equipment, V. 21, No. 6, June 1960, pp. 36-40

Prefabricated shoring systems can save costs on both large and small jobs. They can be used for beams; arches; and parabolic, spherical, and triangular thin shell domes. The three basic units that make up a system of this type are vertical, scaffold, and horizontal shores, both standard and heavy duty with many modifications of all three being available. The heavy duty vertical shores, which are connected to make a tripod shoring tower, are capable of supporting safely 33 tons at 45 ft. Shoring systems can be bought or rented and are easy to maintain as they only need be cleaned after use, and for this a special solvent is available to remove encrusted concrete.

Dams

Control of cracking in TVA concrete gravity dams

Walter F. Emmons, Olva Lavik, and Poul Hornby, *Proceedings*, ASCE, V. 86, PO 1, Feb. 1960, pp. 11-37
Authors' Summary

All concrete dams of TVA are of the straight gravity type. This paper outlines the methods which have been used to control cracking of mass concrete in four of the largest of these. Results include description of cracks observed during and after construction.

Design

Uniform load on a slab or plate stiffened by beams

M. Holmes, *The Structural Engineer* (London), V. 38, No. 6, June 1960, pp. 196-199
Authors' Summary

Describes a method of analysis for beam and slab (or plate) systems. The theoretical analysis in its general form may be used to solve various loading conditions, but the computational work tends to be excessive. For the particular case of a uniform load over the whole area of the stiffened slab, however, the results of the analysis may be expressed as a set of coefficients the values of which may be represented graphically. For uniform loading, therefore, the method of analysis is suitable for design purposes.

Hardy Cross' moment distribution with consideration of shear distortion (Der Momentenausgleich nach Cross bei Beruecksichtigung der Schubverformung)

F. Resinger, *Der Bauingenieur* (Berlin), V. 34, No. 7, July, 1959, pp. 266-268
Reviewed by Aron L. Mirsky

Develops expressions for stiffness, carry-over factors, and fixed-end moments modified to take effect of shear

deformation into account. Method is extended to apply to continuous trusses (the web members taking the shear, are included: a simple-fixed beam of I section under uniformly distributed load, a three-span continuous girder with unequal span lengths and two adjacent spans loaded, and a two-span continuous symmetrical Warren truss with both spans uniformly loaded.

Analysis of rectangular diagrids by anisotropic plate theory

LESLIE G. JAEGER and ARNOLD W. HENDRY, *Proceedings*, Institution of Civil Engineers (London), V. 13 (Session 1958-59), July 1959, pp. 387-392

Reviewed by ARON L. MIRSKY

Authors derive differential equation for the deflection of the substitute anisotropic plate, in terms of coordinates parallel to the supported edges, and apply it to the study of the "straight" grid (one with the same flexural rigidity per unit length in both diagonal directions, and with the support edges at 45 deg to the diagonal directions). Deflections calculated by this theory are shown to be in satisfactory agreement with deflections obtained experimentally using a model. Detailed treatment of the bending moments in the members of a diagrid is promised for a later paper.

The lamella roof

D. SMOLLETT, *The Engineer*, (London), V. 209, No. 5427, Jan. 29, 1960, pp. 173-177

Reviewed by ARON L. MIRSKY

After a brief description of the lamella roof, its history, and the classical (albeit approximate) method of analysis, author explains the rigorous method of analysis he has developed, gives an example of its use (for a timber lamella, but with its application to lamellas of concrete and of other materials duly noted), and compares results of the rigorous and the approximate analyses. Author's method consists basically of displacing any one of the nodes formed by the intersection of the lamella members, setting up simultaneous equations each expressing one of the external forces or moments acting at a node in terms of the movements and rotations of all the nodes, and solving these equations by matrix inversion (using electronic computers) to give the displacements in terms of the known forces and moments; the stresses are then obtained from the displacements.

Stresses in foundation rafts. I

DERYCK N. DEG. ALLEN and ROY T. SEVERN, *Proceedings*, Institution of Civil Engineers (London), V. 15, Session 1959-60, Jan 1960, pp. 35-48

Reviewed by ARON L. MIRSKY

Theory and application of a simple relaxation method of calculating the transverse deflections of concrete raft foundations for buildings, from which the moments and forces can be found. Boundary conditions depend on presence or absence of a wall; method is stated to be particularly suited to evaluating the conditions at the foot of a stiff wall with uniform foundation settlement along its length. Solutions for two cases involving an L-shaped slab are given, (1) with complete external walls, and (2) with two internal walls intersecting at the re-entrant corner; two other cases, (3) with walls along two long sides of the L, and (4) with no walls, will be treated in a subsequent paper.

Computation of northlight-shells by beam method considering the transverse deformation (Die Berechnung von Shedschalen nach dem Balkenverfahren unter Berucksichtigung der Querverformung)

W. MANN, *Beton und Stahlbetonbau* (Berlin), V. 55, No. 33, Mar. 1960, pp 64-68

Reviewed by RUDOLPH SZILARD

Simple beam theory for shells gives a quick result which can be used in design of the shells are long enough. In spite of the fact that extensive tables are available, which facilitate the work of the designer considerably if he uses the rigorous method, the beam method can give relatively good answers if it is improved. In general

the longitudinal stresses are in good agreement, but the transverse moments create considerable differences between the exact and approximate as usual). Three numerical examples methods. The effect of the transverse moments can be considered in the same way as the relative displacements in case of statically indeterminate structures. This small additional work corrects the final result considerably.

On the problem of reliable design of cement-silos (Zur Frage der sicheren Bemessung von Zement-Silos)

F. Leonhart, K. Boll, E. Speidel, *Beton und Stahlbetronbau* (Berlin), V. 55, No. 3, Mar. 1960, pp 49-58

Reviewed by Rudolph Szilard

In many cases damage has occurred in cement silo structures. This fact brought up the necessity of the revision of the presently used design methods. The investigation is based on theoretical and experimental studies. The maximum pressure is a function of the following factors: specific weight of the cement, dimensions of the silo, location and distribution of air-intake ducts, smoothness of the silo wall. The different pressures recommended by various authors are compared and methods for repairing damaged walls are illustrated. The reviewer recommends a reading of this interesting article to all designers engaged in this type of design.

Simplified analysis of flattened arches of parabolic form (Calcul simplifie d'arcs surbaisses de forme parabolique)

R. Masse, *Le Génie Civil* (Paris), V. 136, No. 10, May 15, 1959, pp. 222-223

Reviewed by Aron L. Mirsky

Formulas previously developed by author and applied to analysis of semicircular arches (see following reviews) are here used to develop simple expressions for the redundant reaction components of parabolic arches, for three cases: (1) two-hinged arch, single concentrated load applied at center of span; (2) two-hinged arch, load uniformly distributed along the span; and (3) hingeless arch, loaded as in (2). Study is restricted to symmetrical arches with supports at same elevation and with constant cross-section.

Simplified analyses of arches of circular and parabolic form (Calculs simplifies d'arcs de forme circulaire et parabolique)

R. Masse, *Le Génie Civil* (Paris), V. 134, No. 20, Oct. 15, 1957, pp. 423-424

Reviewed by Aron L. Mirsky

Maxwell-Mohr methods are applied to evaluation of horizontal reaction components in two-hinged arches with reactions at same level. Assuming constant E and I, simple integration gives solution in closed form.

Simplified analyses of circular arches (Calculs simplifies des arcs circulaires)

R. Masse, *Le Génie Civil* (Paris), V. 135, No. 3, Feb. 1, 1959, pp. 64-66

Reviewed by Aron L. Mirsky

Method previously applied to two-hinged arches (see preceding review) is here extended to hingeless arches of circular form, particularly to closed rings. Three cases of loading are studied: (1) two equal and symmertically placed vertical loads at top and bottom; (2) two equal and symmetrically placed radial loads either side of the vertical diameter; and (3) two equal, opposite, and collinear vertical loads placed to one side of the vertical diameter. Wall thickness is assumed constant throughout.

Analysis of roadway slabs supported by transverse beams (Zur Berechnung von Fahrbahnplatte auf Nachgiebigen Quertragern)

Heinrich Bechert, *Beton und Stahlbetonbau* (Berlin), V. 54, No. 12, Dec. 1959, pp. 296-302

Reviewed by Rudolph Szilard

General methods for analyzing continuous roadway slabs supported by transverse beams are based on the conventional rigid support assumption. Taking into account the flexibility of the supports solution for the negative

moments has been worked out by different authors. This paper deals with the determination of the positive moments. The maximum positive moments depend on the stiffness of the supporting transverse beam. The influence of the stiffness on the moment is large. The "effective width" of the roadway slab increases the stiffness. Equations for the positive moments due to the deadloads are derived based on Girkman's solution of the differential equation of the slab. For moving concentrated load the method of singularity is used. The effect of prestressing has been treated.

Materials

Studies on the use of portland blast-furnace slag cement (in Japanese)

TAKAKAZU MARYUYASU, SHUNICHI MIZUNO, and KASUSUKE OKBAYASHI, *Transactions*, Japan Society of Civil Engineers (Tokyo), V. 21, No. 65, Extra Paper 3-1, Nov. 1959
Reviewed by KIYOSHI OKADA

Describes studies made on production, handling, and curing of concrete made from portland blast-furnace cement. Concludes that for slag-cement concrete; (1) the water requirement to produce a given consistency is less; (2) a larger amount of air-entraining agent is required to obtain desired air content, (3) more precaution is necessary in curing, especially at earlier ages, because of slower hardening of concrete and; (4) bleeding is less.

Hydrothermal reactions in the lime-rich part of the system CaO-SiO_2-H_2O

H. G. MIDGLEY and S. K. CHOPRA, *Magazine of Concrete Research* (London), V. 12, No. 34, Mar. 1960, pp. 19-26
AUTHORS' SUMMARY

The mineralogy of autoclaved products from various forms of calcium silicate and from lime and silica has been investigated. The results are as follows. With a CaO/SiO_2 ratio of 1, the product is always well crystallized tobermorite. With a CaO/SiO_2 ratio of 2, the product varies: starting with lime and silica, dicalcium silicate alpha-hydrate is the first product; this eventually changes into dicalcium silicate beta-hydrate, the stable phase; starting with calcium silicate hydrate 1 and lime, the first product comprises Flint's CSH(A) and dicalcium silicate alpha-hydrate, and it seems likely that the Flint's CSH(A) will eventually convert to dicalcium silicate alpha-hydrate and the dicalcium silicate alpha-hydrate to dicalcium silicate beta-hydrate; if one starts with β-$2CaO{\cdot}SiO_2$, the product in the first place is dicalcium silicate gamma-hydrate; in the length of processing used in the experiments reported in this paper, no other phase was found, but it is considered that dicalcium silicate gamma-hydrate is only metastable.

With a CaO/SiO_2 ratio of 3, again the products vary with the starting minerals: with mixes of lime and silica and with mixes of $2CaO{\cdot}SiO_2$ and lime, the first product is dicalcium silicate alpha-hydrate which is eventually converted to the beta-hydrate; with mixes of calcium silicate hydrate (I) and lime, the product is always Flint's CSH(A) with or without some dicalcium silicate alpha-hydrate; with $3CaO{\cdot}SiO_2$ as the starting material, the product is always $3CaO{\cdot}SiO_2{\cdot}1\frac{1}{2}H_2O$.

It is clear from this work that the form of the starting material has a considerable influence on the nature of the metastable phases produced, and also that, in this part of the system CaO-SiO_2-H_2O, even prolonged processing up to 90 days is insufficient to produce true equilibrium phases.

Studies of limestone aggregates by fluid-flow methods

W. L. DOLCH, *Proceedings*, ASTM, V. 59, 1959, pp. 1204-1215 (including discussion)
AUTHOR'S SUMMARY

Fluid-flow measurements were made in an attempt to learn more about the pore characteristics of coarse aggregates and their influence on the freezing and thawing durability of

concrete. Determinations of density, porosity, absorption, degree of saturation, specific surface area, capillary absorptivity, permeability, and tortuosity factor were made on four Indiana limestones with both good and poor field and laboratory durability records. Absorptivity was found to be better than permeability as an index of surface area. The poor stones had consistently larger values of porosity, absorption, specific surface, permeability, absorptivity, ratio of absorptivity to permeability, and coefficient of rate of saturation increase than those for the more durable materials.

Acetate processed portland cement

B. E. Kester, *Proceedings*, ASTM, V. 59, 1959, pp. 1111-1119 (including discussion)

Author's Summary

To overcome the consolidation of dry portland cement that occurs in silos and bulk hopper cars and is commonly termed "pack set," a study was initiated to obtain free-flowing properties of the dry cement.

It was known that certain fatty acids or triglycerides of animal fats would in fact produce free-flowing cement. However, such additives impart water repellency to the cement and entrain air in mortars or concrete. A study of organic acids disclosed that acetic acid had the unique property of creating a free-flowing dry cement and did not, in any measurable manner, alter other properties of the cement. The addition of calcium acetate, either mixed or interground with the cement, was not effective. The reaction of acetic acid with portland cement produces calcium acetate, but since the reaction is dependent on ionization requiring the presence of water, and water in the vapor state is dispersed throughout the cement, the reaction forms a completely dispersed and surface coating of calcium acetate of the cement grains.

It was further found that soluble salts of acetic acid, which had a greater solubility than calcium acetate, were also effective in creating free-flowing cement.

Acetate salts of lower solubility than calcium acetate are without effect on the free-flowing properties of cement on intergrinding.

With the process for creating free-flowing cement established and with positive data that in other properties the cement was unchanged, attention centered on test procedures to evaluate degree of free-flowing. A simple, qualitative test comparison between treated and untreated cement clearly demonstrates the effectiveness.

Effectiveness of mineral admixtures in preventing excessive expansion of concrete due to alkali-aggregate reaction

Leonard Pepper and Bryant Mather, *Proceedings*, ASTM, V. 50, 1959, pp. 1178-1203 (including discussion).

Authors' Summary

Twenty materials, representing eight different classes of mineral admixtures, were evaluated, using both chemical and mortar-bar test methods, for their effectiveness in preventing excessive expansion of concrete due to alkali-aggregate reaction. It was found that the chemical tests cannot be used with reliance to evaluate effectiveness. Each of the replacement materials evaluated will prevent excessive expansion if a sufficient quantity is used. Correlations were found between effectiveness and: fineness, dissolved silica, and percentage of alkali retained by reaction product.

Note

Copies of books and articles reviewed are not available from ACI. Available addresses of publishers are listed in the June "Current Reviews" each year. In some cases ACI can furnish addresses of publications added later.

Five of the materials tested (fly ash, tuff, calcined shale, calcined diatomite, and uncalcined diatomite) showed a reduction in alkalinity of 40 percent or more when tested by the quick chemical test. All of these except the fly ash met the requirement proposed by Moran and Gilliland for the relationship between reduction in alkalinity and silica solubility.

Six of the materials tested (two slags, a fly ash, a pumicite, and two calcined shales) reduced mortar-bar expansion at least 75 percent with high-alkali cement and Pyrex glass aggregates when used as 50, 45, 35, and 30 percent replacement of the cement.

Calculations were made that suggest that the minimum quantity of each material required for effective prevention of excessive expansion ranged from 10 percent for the synthetic silica glass to 45 percent for one of the slags. By groups, these calculated minimum percentages were: calcined shales, 19 to 29; uncalcined diatomite, 22; volcanic glasses, 32 to 36; slags, 30 to 45; and fly ashes, 40 to 44.

Pavements

No-joint paving — the '60 way

JOHN REHFIELD, *Construction Equipment*, V. 21, No. 3, Mar. 1960, pp. 42-47

With over 3,000,000 lb of reinforcing bars to place in a 6-mile stretch of highway near Waco, Tex., the contractor come up with three ideas to cut costs on this 8-in. slab: (1) built-in chairs to keep reinforcing in exact center of slab, (2) vibrated spreading, and (3) jointless center joints.

Dynamic testing of pavements

W. HEUKELOM and C. R. FOSTER, *Proceedings*, ASCE, V. 86, SM 1, Feb. 1960, pp. 1-27

AUTHORS' SUMMARY

Each time a vehicle passes over a pavement the surface is deflected and rebounds, creating temporary strain conditions for 0.01 to 0.1 sec. Information about these strain conditions can be obtained by observing the movements of the pavement under similar dynamic loading conditions. Methods used for the dynamic investigation of pavements, base courses, and subgrades are described.

Expanded design charts for concrete pavement

FRANK MCCULLOUGH and WILLIAM B. LEDBETTER, *Civil Engineering*, V. 30, No. 1, Jan. 1960, pp. 77-78

The design charts are based on empirical formulas for unreinforced joined pavement with protected and unprotected corners. As these charts are based on a modulus of elasticity of 4,000,000 psi, and since other moduli have been encountered, the authors have presented two graphs to enable designers to work with concretes whose modulus of elasticity is different from that given in the original chart.

Prestressed Concrete

Examination of the stresses in the anchorage zone of a post-tensioned prestressed concrete beam

D. J. DOUGLAS and N. S. TRAHAIR, *Magazine of Concrete Research* (London), V. 12, No. 34, Mar. 1960, pp. 9-18

AUTHORS' SUMMARY

The work described is the result of investigations carried out in an attempt to determine the effect of high concentrations of stress in the anchorage zone of a prestressed concrete beam. The problem was idealized as that of the stress distribution due to the application of a circular concentrated load acting at one end of a concrete cylinder, the theoretical analysis and experimental procedure being in this way simplified. A mathematical analysis of a three-dimensional case to determine the stresses throughout the anchorage zone is described and the results are compared with those of other writers. In a series of tests designed to verify the theoretical analysis, the loads required to cause failure were much greater than those predicted on the basis of current theories of failure and on the assumption of

elastic behavior. Further experimental work involving measurement by electrical resistance strain gages of the surface strains showed good agreement with strains predicted theoretically. Attention is called to the fact that the tensile strains measured corresponded to stresses of approximately four times the ultimate tensile stress of the concrete as determined by the Brazilian method of testing.

Some prestressing problems of plate bridges

S. KAUFMAN and J. GLOMB, *Bulletin de l'Academie Polonaise des Sciences* (Warsaw), V. 7, No. 5, 1959, pp. 353-358
APPLIED MECHANICS REVIEWS V. 13, No. 6, June 1960

The moment in the longitudinal direction of a slab (plate) bridge due to a concentrated load occupying various positions at midspan will be maximum when the load is applied at either longitudinal edge and minimum when applied at the center of the slab. This variation of moment can be accommodated (1) by suitable layout of longitudinal prestressing or (2) by tracing the transverse cables in such a way as to obtain uniform distribution of longitudinal prestressing. The second case is considered in this paper. Equations for determining the prestressing forces and eccentricity of the transverse cables are derived and the mathematics is fairly simple to follow.

Study of the grout for prestressed concrete

I. LYSE and R. JOHANSEN, *RILEM Bulletin* (Paris), No. 4, New Series, Oct. 1959, pp. 59-65
AUTHORS' SUMMARY

Paper first considers the various requirements for a proper grout for prestressed concrete structures, placing particular emphasis on the frost resistance of the grout. A description is given of an investigation of the frost resistance and other qualities of the grouts carried out at the Technical University of Norway. The tests showed that ordinary neat cement grout would crack and also cause the surrounding concrete to crack when subjected to freezing temperatures immediately on grouting. The addition of air-entraining agents did not change essentially the behavior frozen immediately, but after a few days of curing an air entrainment of 8 to 12 percent would remove the danger of cracking. The air entrainment also reduced the tendency of bleeding of the grout. Introduction of special plasticizers practically eliminated bleeding. Neat cement grout without the addition of air-entraining agents may cause cracking of the grouted structure even after several days of curing before the exposure to freezing temperatures. The necessity of proper methods of tests of grouts for prestressed concrete structures is emphasized.

Properties of Concrete

Shrinkage and creep of specimens of thin section

C. DE LA PENA, *RILEM Bulletin* (Paris), No. 3, New Series, July 1959, pp. 60-70
AUTHOR'S SUMMARY

The testing method used makes it possible to avoid the effects produced on usual test pieces by the gradient of shrinkage between the core of the test piece and the surface.

Shrinkage and creep tests were carried out on mortar test pieces having the form of a tube—length 100 mm, inside diameter 46 mm, thickness in center of tube 2 mm. With this type of test piece the value of the shrinkage and final creep can become known more quickly than with the usual test pieces. Altogether, 216 test pieces were made for these tests.

The author reached the following conclusions: (1) The shortenings are not proportional to the relative degree of hygrometry of the air in which the test pieces are cured. (2) The shortenings are not proportional to the loads applied. For an increase of 20 percent of the applied load, the deformation increases by 50 percent. (3) For each mix, the replacement of

a fraction of sand by a fraction of cement produces the same increase in shortening (approximately 12 percent). (4) The shortening of the test pieces tested in moist air (expressed in percentage of the shortening of the test pieces tested in dry air) increases when the applied load increases. (5) For the batchings least rich in cement (if shortenings are expressed in relation to shortenings of the mortars with the richest batching) the weaker the load the greater the shortening. (6) The shortening of each mix (expressed in percentage of the shortening obtained under the driest curing conditions) seems to be practically the same, provided the hygrometry remains constant.

Recovery of creep and observations on the mechanism of creep of concrete

A. M. Neville, *Applied Scientific Research* (The Hague), Sec. A, V. 9, No. 1, 1959, pp. 71-84

Reviewed by Aron L. Mirsky

Test data are given for creep recovery and sand-cement mortar specimens made with 16 different cements of six different types and stored under wet or dry conditions. Results indicate that whereas creep is dependent on the mortar strength (as influenced by the cement), this is *not* true of creep recovery, which appears to be governed by the rigidity of the cement paste as a whole (which varies but little between different pastes).

Shrinkage and loss of weight of the mortar specimens were also investigated. Results appear to indicate that in the early life of the mortar a loss of water occurs without a corresponding decrease in the volume of the specimen. The loss of weight of loaded specimens is the same as the loss of unloaded specimens over the same period of time; the behavior of dry and of wet specimens is similar.

Author suggests that the mechanism of creep is related to the movement of zeolitic water from the calcium silicate hydrate into the capillary pores in the gel, and that the rigidity of the cement paste as a whole may be the governing factor in rendering the creep movement non-reversible.

Strength tests of low-slump concrete

D. L. Bloem, *Cement, Lime, Gravel* (Birmingham, England), V. 34, No. 1, 1959, pp. 21-22; *Quarry Managers' Journal*, V. 43, No. 3, 1959, pp. 109-110

Road Abstracts
V. 27, No. 3, Mar. 1960

Plastic concretes (2- to 3-in. slump) were compared with very dry concretes having substantially no slump; three cement factors (6, 7½, and 9 sacks per cu yd) were used with three coarse aggregates representing a broad range in strength-producing properties. Beams and cylinders were tested after 28 days of moist curing; the dry concrete was vibrated instead of being rodded. Results are presented in graphs and tables. The use of dry concrete produced on the average an advantage of about 1000 psi compressive strength and about 80 psi flexural strength for a given cement factor and coarse aggregate. These differences could not be due entirely to the effect of water cement ratio and vibration. It is therebe due entirely to the effect of water-cement ratio and air content, dry mixes give higher strengths than concretes having slumps of 2 to 3 in.

Method for determining water-cement ratio of freshly mixed concrete (in Japanese)

Shunichi Mizuno, *Journal*, Japan Society of Civil Engineers (Tokyo), V. 44, No. 10, Oct. 1959, pp. 1-7

Reviewed by Kiyoshi Okada

A simple method is proposed for determining water-cement ratio of fresh concrete. A given quantity of mortar, which is obtained by screening from the sample of concrete, is dispersed by adding water. The cement content of the mixture is measured by the hydrometer, and the water content by the heating method, respectively. Also discusses the effects of various factors on the accuracy of the measurements.

Effect of length to diameter ratio on compressive strength — An ASTM cooperative investigation

CLYDE E. KESLER, *Proceedings*, ASTM, V. 59, 1959, pp. 1216-1229 (including discussion)
AUTHOR'S SUMMARY

Ten laboratories participated in a program to determine the effect of the length to diameter ratio on the apparent compressive strength of concrete. The concrete strength, type of aggregate, specimen size, and curing were variables. At present, ASTM provides only one set of factors for "correcting" strengths obtained from specimens which have a length to diameter ratio less than the standard, 2. The cooperative study clearly shows that the correction factors are dependent on strength and also that lightweight aggregates require different factors than do normal-weight concretes.

On extensibility of fresh concrete under slowly increasing tensile loads

K. OHNO and T. SHIBATA, *RILEM Bulletin* (Paris), No. 4, New Series, Oct. 1959, pp. 24-31
AUTHORS' SUMMARY

Presents some results of experimental investigations on the extensibility of fresh concrete under increasing tensile loads when the speed of loading is varied. The size of specimen was 4 x 4 x 20 cm. The tests were carried out in two series. One concrete mixture ($w/c = 0.60$, proportion = 1:2.85:2.55, slump = about 10 cm) was adopted in both cases.

When the concrete is stored in dry condition and is sustained under increasing tensile stresses from the seventh day after the casting, the magnitude of the tensile strain increases almost inversely proportionally to the loading speed. The largest extensibility can be obtained for the lowest loading speed, and the maximum total strain becomes about four times that of usual short time test. Comparison of each strain corresponding to a load increment shows that the maximum strain appears in the first step of loading, that is, in the case of youngest concrete.

When the application of tensile load is started while the concrete is still wet, the maximum extensibility appears under a certain loading speed which is not so slow. The reason may be that a wet concrete has more extensibility than a dry concrete under tensile stresses. If the loading speed is too slow, the concrete may dry before the stress becomes high, and it may be understood that the strain does not increase so much even if the stress is increased sufficiently high later and the final strain does not show the highest value.

Shielding concrete of the research reactor FR 2 at Karlsruhe (Der Abschirmbeton des Karsruher Forschungreaktors FR 2)

ALFRED BAUER and JURGEN SEETZEN, *Beton und Stahlbetonbau* (Berlin), V. 54, No. 12, Dec. 1959, pp. 281-293
Reviewed by RUDOLPH SZILARD

Deals with the preliminary work and research for the first German nuclear reactor FR 2 in Karlsruhe using concrete shielding. First the general technical and physical requirements of a reactor shielding are described, later the specific requirements of the reactor at Karlsruhe are detailed. In general, two layers of shielding are required: thermal and biological. The thermal shield enclosing the reactor is of structural steel and lead to reduce the high temperature produced by thermal radiation. The second shielding layer, which encloses the first, starts at the point where the concrete is able to sustain the high temperature effects. Purpose of this second layer (biological shielding) is to reduce and absorb the neutron radiation produced by the nuclear reaction to a level which can be tolerated by human beings without harmful effects. For aggregate, magnetite has been selected; there was no reason for using other than regular portland cement. A unique construction method has been used to achieve maximum density in the concrete. This method, called Auspressverfahren, consists of

the following steps: First the rough aggregate is placed into the formwork and then the voids are filled in by high-pressure grouting. A series of tests have been performed by the Technical Institute of Karlsruhe to check the concrete properties.

Relation between ultrasonic pulse velocity and the compressive strength of concretes having the same workability but different mix proportions

M. F. KAPLAN, *Magazine of Concrete Research* (London), V. 12, No. 34, Mar. 1960, pp. 3-8

AUTHOR'S SUMMARY

Experiments have been made to investigate whether, for concrete of a given workability, the relation between ultrasonic pulse velocity and compressive strength is independent of variations in age and in mix proportions (i.e., aggregate-cement and water-cement ratios).

It is concluded that, for concrete made of the same type of material, curves relating pulse velocity to compressive strength may be drawn for concrete of the same workability but different mix proportions, provided that the tests are done at the same age. When tests are done at different ages the relation is dependent on both mix proportions and age. This dependence, is, however, not so apparent at low compressive strengths.

An approach to the problem of evaluation of plasticity and workability of pastes and mortars

A. D. CONROW, *Proceedings*, ASTM, V. 59, 1959, pp. 1075-1090 (including discussion)

AUTHOR'S SUMMARY

In this investigation an attempt was made to develop a means for evaluating some of the properties of pastes and mortars that influence their plasticity and workability. Measurements were made of the rates of deformation of hemispherical specimens of pastes and mortars when tested on the flow table. The deformation was recorded for each drop of the table and the change in percent flow was used as a measure of the deformation.

It was found that the deformation was apparently related to the energy of the drop (weight of specimen times height of drop) by an equation of the type $F = AE^b$, where F = percent flow, E = energy applied and A and b are constants which appear to be related to ease of deformation and to cohesiveness (ability of the elements of the mass to cling together when the mass is deformed). The significance of the constants A and b, which are evaluated from the test results, is discussed.

Study of the creep of concrete

M. MAMILLAN, *RILEM Bulletin* (Paris), No. 3, New Series, July 1959, pp. 15-31

AUTHOR'S SUMMARY

The study presented in this report had reference to continuous creep, the load being applied along the longitudinal axis of the test piece. The deformations produced were measured, in the direction of the force, on two opposite surfaces of the test piece.

Various factors were studied: duration of the load, magnitude of the stress, age at which load is applied, hygrometric curing conditions, effect of unloading, mechanical characteristics of the concrete.

The attempt to find a test method for creep under tension of test pieces of pure cement paste is described.

It seems that the creep of test pieces under compression tends toward a final value that may be estimated to within 15 percent when the evolution of the phenomenon has been observed for 6 months. The creep is proportional to the stress applied, whatever be the age at loading or the hygrometry, provided 50 percent of the rupture stress is not exceeded. The age at loading in dry air is of importance only if it occurs before 28 days. The hygrometric conditions are of capital importance for the evolution of the phenomenon. Curing in water, other things being equal, makes it possible to obtain a

creep amounting to only one-fourth that obtained by curing in air at 50 percent humidity.

New method to study the early volume changes in neat cement paste

M. Del Campo, *RILEM Bulletin* (Paris), No. 4, New Series, Oct. 1959, pp. 18-23
Author's Summary

Presents a new method for the study of the variations in volume of pure cement paste, immediately after depositing, during setting and initial hardening. The report gives the initial results obtained by a dilatometer that isolates the paste, to avoid the effects of evaporation, of the propagation of humidity and of temperature.

The test pieces used were made with a portland slag cement. The following variables were studied, in particular: water-cement ratio; fineness of the slag; and percentage of SO_3.

The water-cement ratio seems to be the factor, among all these variables, that has the most influence on the variations in volume during the first hours. Measurements were made, besides, of the deformations during the first 24 hr for ten portland cements of different qualities.

Frost-corrosion resistance of hydrotechnical concretes made up of certain Polish cements (in Polish)

W. Gaca, *Archiwum Hydrotechniki* (Warsaw), No. 1, 1959, pp. 57-89
Polish Technical Abstracts No. 1 (37), 1960

Research was carried out with a view to establishing the ingredients of three types of concrete intended to test elements of a hydrotechnical structure in Gdansk Bay; it was further desired to prove which of the three kinds of cement tested was the most resistant to the joint action of physical (frost) and chemical corrosion, as also to show which ingredient makes the concrete corrosion resistant.

The following conclusions are drawn: (1) To carry out tests for frost-corrosion resistance over a period of 7 days, and to select the most corrosion resistant concrete, it is sufficient to perform 25 freezing and thawing cycles. Tests with samples over a period of 28 days, however, require at least 100 cycles when it is desired to establish the influence of the slag content in slag cement or to determine the influence of the value of the aluminum modulus in portland cements on their frost-corrosion resistance.

In the shorter test, the grade of cement or, strictly speaking, the grade of concrete has the predominant influence on the frost-corrosion resistance. The tests carried out show that, regardless of the cement grade, the concrete having a minimum compressive stress of 250 kg per sq cm after 7 days is fully resistant to marine corrosion in the Gdansk Bay region. When specific marine conditions are taken into consideration, it has to be concluded that hydrotechnical concretes are, as a rule, subjected to an intermediate action by sea water, and hence their strength after 28 days does not represent a satisfactory criterion for the estimation of the frost-corrosion resistance.

(2) To speed up the process of corrosion of the samples examined, the sea water employed should be of fivefold concentration, and samples should be fully immersed in this solution. Sea water of higher concentration is not recommended since it weakens the freezing action, which is an agent of the mechanical corrosion. A solution of a higher concentration does not freeze completely, hence the bursting action, caused by the formation of ice in pores of concrete, does not take place.

(3) It is not recommended, regardless of the utilization time of hydrotechnical structures, to use calcium chloride admixtures together with post-alcoholic distillate from sulfite lye for concretes made of portland cements, owing to the negative influence of those admixtures upon the frost-corrosion resistance of the said concretes. Taking into account the

positive influence of the above admixtures on the chemical (sulfate) resistance cement slurries, it must be observed that the noxiousness of the chemical attack cannot be compared to the noxiousness of corrosion caused by frost. The above admixtures, used jointly, add considerably to the increase in frost-corrosion resistance solely as regards concretes made of slag cement of high slag content, (approximately 60 percent), and at the beginning of the utilization period only. The positive influence of these admixtures in the slag cement ceases after 28 days.

Structural Research

Notes on the testing of reinforced concrete model shells

G. R. MITCHELL, *Indian Concrete Journal* (Bombay), V. 33, No. 12, Dec. 1959, pp. 435-437
AUTHOR'S SUMMARY

This paper mentions some of the factors which need to be taken into consideration when undertaking tests on model shells and some of the practical difficulties which may be encountered.

Splicing of cold-drawn reinforcing bars with anchoring rings (in Swedish)

CARL FORSSELL, *Nordisk Betong* (Stockholm), V. 4, No. 1, 1960, pp. 47-50
Reviewed by MARGARET CORBIN

It is suggested that "HJS 70" reinforcing bars should be spliced by means of screwed splices, flat bar cross piece splices, stirrup cross piece splices, or lapped splices. Splices of the last-mentioned type have been tested and the results of these tests are shown. The cube strength of the concrete used in the tests was 97 kg per sq cm, and the stress in the reinforcement at splice failure was 4400 kg per sk cm. If use is made of Swedish standard grade K 250 concrete, and if the anchoring rings are placed in accordance with the relevant Swedish standard specifications, then this corresponds to splicing to full strength. Thus, all the above-mentioned splices render unnecessary the restrictions imposed on splicing by the provisional Swedish standard specifications for cold-worked reinforcing bars with anchorage rings which are in force at the present time.

Influence-fields for oblique-angled, curved, three-span plates with variable plate thickness (Einflussflachen fur eine schiefwinklige, gekrummte Dreifeldplatte mit veranderlicher Plattendicke)

K. HEINISCH, *Beton und Stahlbetonbau* (Berlin), V. 55, No. 3, Mar. 1960, pp. 58-64
Reviewed by RUDOLPH SZILARD

It is a very complex problem to determine the moment influence fields, etc., using analytical methods. The Technical University of Dresden used small scale models for this purpose. The models were made to $\frac{1}{25}$ scale, the strains have been measured by electric gage under different concentrated loads. From the measurements the influence fields were calculated.

Investigation of the flexural and shearing capacity of reinforced concrete beams

JOHN E. BREEN and ADRIAN PAUW, *University of Missouri Bulletin* (Engineering Experiment Station Series No. 49), V. 61, No. 33, 1960, 84 pp.

Report of a research program to study design and economic criteria for precast bridge units. The initial stage of the program included a comprehensive study of typical sections under simulated loading conditions to determine both their behavior at nominal working loads and their ultimate capacities with respect to flexural or shear failure.

To provide a field check for the instrumentation devices developed for measuring strains and deflections, as well as to provide fundamental data for the proper planning of the main test program, a series of tests was run on 23 reinforced concrete beams with rectangular cross sections. This report presents the results of these tests and a survey of related engineering literature.

ACI NEWS LETTER

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memo from the President:

WHAT NEXT?

ON ANY EXTENDED JOURNEY it is good occasionally to reorient and to chart the course ahead. ACI did this in 1948 when it adopted a committee report of eight long-range goals of the Institute. Ten years later, at the 1958 annual convention in Chicago, President Walter H. Price evaluated the progress made toward those goals. As would be expected, he found that some had been achieved, some had been approached, and some were still ahead. In any event, the definition of purpose and the attention directed toward that purpose undoubtedly had important influences on ACI progress during the intervening 10 years.

Now actively at work is another committee on aims and objectives, headed by Walter J. McCoy and with members Douglas McHenry, Walter H. Price, Paul F. Rice, Charles H. Scholer, J. Neils Thompson, and Stanton Walker. It would welcome suggestions from ACI members, addressed to the chairman. We may expect a report soon, but in the meantime it would be worthwhile for each member independently to consider best ways and means of furthering the progress of the Institute.

What *is* good for the Institute? Whatever is good for its members, and whatever is good for concrete.

As to members, the more the better. Additional members bring new viewpoints, contributions to committee effort, and dues to share

in the support of our expanding program. Present members can interest their friends who are concerned with concrete, and by recommending membership can benefit both the Institute and the individual.

More important, each individual member should take advantange of ACI resources to advance himself in his work, whatever it is. From its publications, meetings, and committee contacts he can learn much as well as contribute. Essentially he is attending a college of concrete.

Concrete means many things to many people, and its making requires concerted effort of many people, from designer to construction laborer. Results of the team effort are hindered by faulty performance of any of its members. An individual's goal should be to see that *his* bit is done well; that the finished product is worthy of the best thought and action of the construction industry; and that still better and more economical materials and methods are developed.

The committee report will be submitted soon, and we await it with interest. It will be specific. These general remarks are intended to supplement it by reminding each member that his individual goals are important parts of the goals of the Institute as a whole.

Joel W. Kelly
President

55-Year Index soon a reality

Early this month the cumulative 55-Year Index of the ACI *Proceedings* and JOURNAL will go to the printer. The index, covering the years 1905-1959, replaces the 20-Year Index, 1929-1949; the 5-Year Supplemental Index, 1950-1954; and the yearly indexes published since 1954. It will include, in addition, all *Proceedings* material heretofore not indexed (1905-1928); thus providing in one convenient book reference to nearly every significant development in the concrete industry in this century.

The book will be handsomely bound in a hard, durable cover and assembled so that it will lay flat on the desk when open. It will contain about 18,000 references, which have been consolidated for easy use, and is liberally cross referenced. In addition, to further aid the searcher in finding exactly the material needed, it will contain a section of brief synopses of every paper published in the ACI JOURNAL since it first appeared in 1929, similar to that found to be so useful in the 20-Year Index.

This long-awaited index completes the program begun in 1947 with the 10-Year Index, 1937-1947. Stage two of the program was the 20-Year Index, of which over 16,000 copies were distributed.

As in past ACI indexes, the 55-Year Index will reference papers by title, author, and subjects. The main portions of references are to be printed in boldface type to aid the user in finding that material he desires.

ACI members and JOURNAL subscribers will receive by mail an order blank that will allow them to get the index at a special introductory price.

13th Regional Meeting
October 31-November 2

ACI Meets in Tucson

THE ACI 13TH REGIONAL MEETING will be held in Tucson, the "sunshine city" of the southwest, October 31 to November 2, with headquarters at the Pioneer Hotel.

Highlights of the 3-day program include joint ACI-ASTM technical sessions, scenic sightseeing tours, and special activities for the ladies, all spiced with true Tucson hospitality.

The technical program opens Tuesday, November 1, featuring a morning session on construction with Frank Stubbs, University of California, Berkeley, as chairman; D. R. Butler, Integrated, Inc., Los Angeles, current president of the ACI Southern California Chapter, will act as chairman of the afternoon session on research in design.

Wednesday, November 2, will feature concurrent technical sessions in the morning: an ACI design session, B. J. Shell, San Xavier Construction Co., Tucson, chairman; and an ASTM research session, John T. Young, Arizona Aggregate Association, Phoenix, chairman. The joint afternoon research session will be chairmanned by Cedric Willson, Texas Industries, Inc., Dallas, Tex.

A joint ACI-ASTM luncheon on the day of the concurrent sessions has been arranged with A. Allan Bates, vice-president, Research and Development, Portland Cement Association, Chicago, currently president of ASTM, as guest speaker. Long active in ACI affairs, Dr. Bates is a member of the Institute Board of Direction.

Added attractions planned by the local committee include a chartered bus trip on Tuesday, November 1 to the "Old Tucson" movie set and the Sonora Southwest Desert Museum followed by a western chuck wagon-style dinner in the evening. The following day a bus trip into Mexico, luncheon at the Caverns dining room in Nogales, Sonora, and a shopping spree for the wives "south of the border" will keep the ladies well entertained. Mrs. Gene M. Nordby is chairman of the ladies committee.

Gene M. Nordby, head, civil engineering department, University of Arizona, is general chairman of the regional meeting with Andrew Ross, vice-chairman and Joseph Toth, treasurer, both of the University of Arizona. Edward Mangotich, Portland Cement Association, a member of the executive planning committee is in charge of exhibits.

Others assisting in arranging the meeting are: B. J. Shell, San Xavier Rock and Sand Co., technical program chairman; Lee Lawrence, civil engineer, finance chairman; Gene Anderson, civil engineer, publicity and sessions chairman; and Andrew Marum, consulting engineer, membership chairman.

Russ Taylor, Taylor-McFarland-Johnson, is in charge of luncheon and banquet details; Maurice LeGaard, field

REMEMBER! You do not have to be a member of ACI or ASTM to attend the regional meeting. All sessions are open to everyone interested in concrete construction, design and research. Bring a friend.

trips; Howard Harrenstien, educational contacts; and K. K. Kienow, registration. The latter three members of the local planning committee are all affiliated with the University of Arizona.

TECHNICAL PROGRAM

TUESDAY MORNING, NOV. 1

Construction Session

Frank Stubbs, Chairman

Missile Base Construction Problems

Lt. Col. G. H. Andrews, Strategic Air Command, U. S. Air Force, Offutt AFB, Omaha, Neb.

Developments in Timber Falsework

George Birkemeier, Timber Structures, Inc., Portland, Ore.

Effects of Recent Mexican Earthquakes on Reinforced Concrete Structures

R. D. de Cossio and Emilio Rosenblueth, University of Mexico, University City, Mexico D.F., Mexico

World's Largest Lift Slab, The Motorola Building, Phoenix

Charles Magadini, consulting engineer, Phoenix, Ariz.

TUESDAY AFTERNOON, NOV. 1

Research in Design Session

D. R. Butler, Chairman

Solution of Structural Problems through Model Analysis

Jack R. Janney, Engineers Collaborative, Chicago

Full Scale Lightweight Experimental Bridge

Truman R. Jones and T. J. Hirsch, Texas Agricultural and Mechanical College, College Station

Failure of Small Reinforced Concrete Beams under Repeated Loads

J. R. Verna and T. C. Stelson, Carnegie Institute of Technology, Pittsburgh

Fatigue Properties of Lightweight Concrete

J. F. McLaughlin, Purdue University, Lafayette, Ind.

CONCURRENT SESSIONS

WEDNESDAY MORNING, NOV. 2

ASTM Research Session

John T. Young, Chairman

Water Vapor Permeability of Concrete

D. F. Griffin, Naval Civil Engineering Laboratory, Port Hueneme, Calif.

Effect of Steam Curing on Important Concrete Properties

Elmo C. Higginson, Bureau of Reclamation, Denver

Concrete Retempering Studies

M. J. Hawkins, Bureau of Reclamation, Ephrata, Wash.

Properties of an Expansive Cement for Chemical Prestress

Alexander Klein, T. Karby, and Milos Polivka, University of California, Berkeley

ACI Design Session

B. J. Shell, Chairman

Prestressed Composite Design in Multistory Building Construction

Harry Edwards, Leap Associates, Lakeland, Fla.

Prestressed Girder Design, Testing, and Construction for Norton Building, Seattle

Arthur R. Anderson, Anderson, Birkeland, and Anderson, Tacoma

Hyperbolic Reinforced Concrete Cooling Towers

Paul Rogers, Paul Rogers and Associates, Chicago

Prismatic Folded Plates — A Design Procedure

A. A. Brielmaier, Washington University, St. Louis

WEDNESDAY NOON, NOV. 2

Joint ACI-ASTM Luncheon

Guest speaker — A. Allan Bates, Portland Cement Association, Chicago

WEDNESDAY AFTERNOON, NOV. 2
Joint ACI-ASTM Research Session
Cedric Willson, Chairman

Heat Resistant Concrete for Missile Bases
Herman G. Protze, materials technologist, Boston

Abnormally Slow-hardening Concrete
L. H. Tuthill, R. F. Adams, S. N. Bailey, and R. W. Smith, California Department of Water Resources, Sacramento

The Use of Epoxy Resins on Highways
R. E. Brown, Shell Chemical Co., New York

Epoxy Resins as Structural Adhesives
J. D. Kriegh and T. C. Ewbank, University of Arizona, Tucson

ACI technical committee appointments

Listed below are committee members who have recently accepted appointment to ACI technical committees. Included are *new* appointments only.

Committee 207, Properties of Mass Concrete

Ralph L. Bloor
International Bank for Reconstruction and Development
Washington, D. C.

G. E. Goodall
Consulting Civil Engineer
Sacramento, Calif.

Committee 213, Properties of Lightweight Aggregates and Lightweight Aggregate Concrete

H. P. Mittet
University of Washington
Seattle, Wash.

Thomas W. Reichard
National Bureau of Standards
Washington, D. C.

Committee 216, Fireproofing or Fire Protection of Structures

Allan H. Stubbs
Fire Prevention Research Institute
Gardena, Calif.

N. G. Zoldners
Canada Department of Mines and Technical Surveys
Ottawa, Ont., Canada

Committee 323—Prestressed Reinforced Concrete—Joint ACI-ASCE

W. Burr Bennett, Jr.
Portland Cement Association
Chicago, Ill.

J. N. Clary
Virginia Department of Highways
Richmond, Va.

Ben C. Gerwick
Ben C. Gerwick, Inc.
San Francisco, Calif.

T. Y. Lin
University of California
Berkeley, Calif.

C. E. Mason
Besser Co.
Alpena, Mich.

Alan H. Mattock
Portland Cement Association
Skokie, Ill.

George S. Pinter
Precrete, Inc.
Corona, N. C.

C. P. Siess
University of Illinois
Urbana, Ill.

W. R. Wilson
The Atchison, Topeka and Santa Fe Railway System
Chicago, Ill.

Committee 331, Structures of Concrete Masonry Units

Samuel Judd
U. S. Bureau of Reclamation
Denver, Colo.

Committee 335, Deflection of Concrete Building Structures

Sabri Sami
West Virginia University
Morgantown, W. Va.

Committee 338, Torsion

Paul Andersen
University of Minnesota
Minneapolis, Minn.

Committee 402, Concrete Floor Finishes

C. B. Elder, Jr.
Trap Rock Material and Engineering Co.
Iron Mountain, Mo.

Committee 505, Design and Construction of Reinforced Concrete Chimneys

L. C. Maugh
University of Michigan
Ann Arbor, Mich.

Committee 612, Recommended Practice for Curing Concrete

E. L. Howard
Pacific Cement and Aggregates, Inc.
San Francisco, Calif.

Joseph J. Waddell
Knoerle, Graef, Bender and Associates, Inc.
Chicago, Ill.

Committee 613, Recommended Practice for Proportioning Concrete Mixes

E. A. Abdun-Nur
Consulting Engineer
Denver, Colo.

George Werner
Bureau of Public Roads
Washington, D. C.

J. E. Gray
National Crushed Stone Association
Washington, D.C.

Karl Nensewitz
Besser Technical Center
Alpena, Mich.

Committee 716, High Pressure Steam Curing

Clyde L. Stewart
Autoclave Building Products Association
Chicago, Ill.

George W. Washa
University of Wisconsin
Madison, Wis.

Roadbuilding costs still dropping

In spite of general economic inflation, the cost of highway construction is dropping slightly according to the Better Highways Information Foundation, Washington, D.C.

The U.S. Bureau of Public Roads has reported that the cost of road work, including projects on the 41,000-mile interstate system, fell another ½ percent during the second quarter of 1960. This followed a more substantial decline of 2.7 percent in the first quarter. Statistics indicate that highway construction costs have been gradually decreasing for the last 3 years.

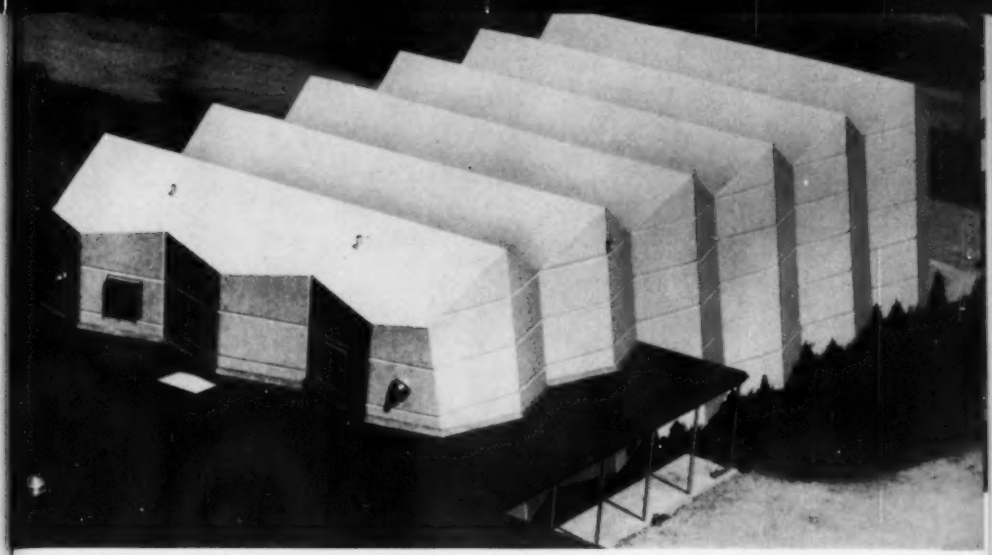
Highway construction costs, using 1946 as a base year, have been up and down for the last 5 years, but currently are only 6 percent more than costs registered during the low period of second-quarter 1955.

Erskine Stewart, acting director of BHIF, attributes the dropping of highway costs to increased efficiency in the industry through bigger machines, newer methods, and intense competition.

Queen's University conducts R/C design symposium

ACI participated in a reinforced concrete design symposium presented by the Department of Civil Engineering, Queen's University, Kingston, Ont., Canada, August 29 through September 2.

The program was planned to give engineers and contractors an opportunity to become familiar with recent developments in the field of load design and the design of reinforced concrete shells. S. D. Lash, head, Department of Civil Engineering, Queen's University, was general chairman of the symposium.



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Architect: Buck Associates, Bartlesville, Oklahoma

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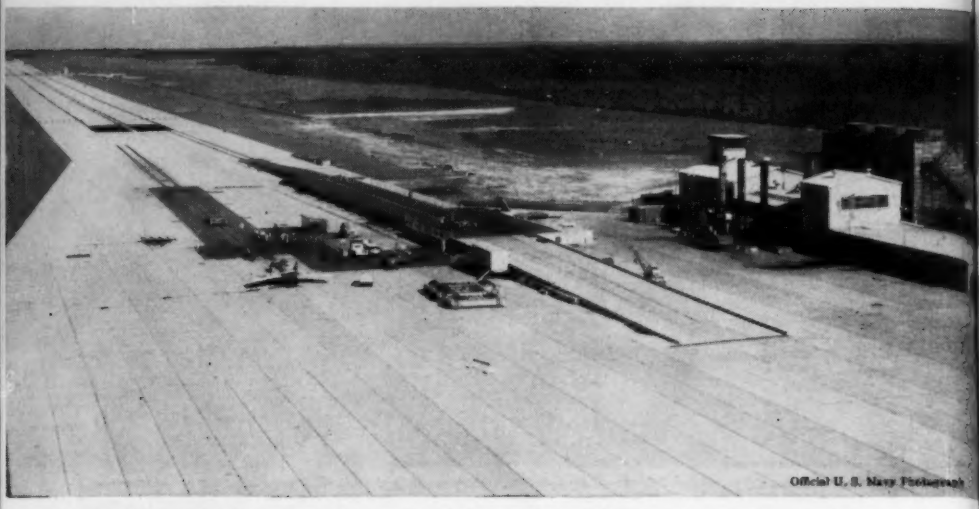

NAVAL AIR TEST FACILITY—Lakehurst Naval Air Station, N. J. Construction Agency: Bureau of Yards and Docks of the Navy Department. Constructed under the direction of the District Public Works Officer, 4th Naval District, Philadelphia, Pa. • Consulting Engineer on Pavements: Louis Berger & Associates, Orange, New Jersey • Contractor: Roscoe Engineering Corporation & Associates, Washington, D. C. • Pavement Sub-Contractor: S. J. Groves & Sons Company, Woodbridge, N. J.

900 psi flexural strength— POZZOLITH helps meet this requirement at The Naval Air Test Facility

The Runway Arrested Landing Site of the Navy's recently completed $23,000,000 test facility required concrete of unusually high quality.

To withstand anticipated stress of jet planes simulating aircraft carrier landings, reinforced concrete having a flexural strength of 900 psi was specified.

Preliminary mix designs—with cement factors ranging up to 8½ sacks per cubic yard and with the water-cement ratio of 4.0 gallons per sack—did not provide a workable mix with the required high flexural strength.

The following mix was then employed:

Cement, Type 1—729 lbs.—(7¼ sks.)
Sand (SSD)—1,000 lbs.
C. A.—Trap Rock—
(Houdaille, Ind., Bound Brook, N.J.)
¾"—895 lbs. 1½"—1342 lbs.
Water (total)—28 gal. (3.62 gal./sk.)
POZZOLITH

This mix was workable and developed a flexural strength of 951 psi in 7 days and 1114 psi in 28 days. Compressive strength averaged 8537 in 28 days.

Because of these high strengths the water-cement ratio was later increased to 3.75 and the cement factor reduced to 7½ bags. Flexural strength of the revised POZZOLITH mix averaged 981 psi in 28 days.

For further details on the performance of POZZOLITH on this and other airfield projects write to—

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*POZZOLITH is a registered trademark of The Master Builders Company for its concrete admixture to reduce water and control entrainment of air and rate of hardening.

Another Prestressed Concrete Milestone

THE $1,968,000 ONEIDA LAKE BRIDGE, a major link in the Empire State Interstate Route 505 between Binghamton, N.Y., and the Canadian border through Watertown, N.Y., is a milestone in prestressed concrete construction. The 470 ft long, three span continuous bridge has a 320 ft central span (center to center of piers) plus two 75 ft end spans and will be one of the longest prestressed structures in the world.

The bridge consists of two separate 45 ft wide parallel structures, 56 ft center to center, carrying three lanes of traffic in each direction.

Each of the two double cantilever prestressed concrete structures is in the shape of a flat arch, with six 146-ft cantilever girders projecting 72 ft beyond the piers on each end. Between them, five 231 ft long, simply supported girders are hung like interlocking fingers. These are T-shaped. After placement, the tees were joined to the cantilever girders by a 7-in. concrete deck closure strip and post-tensioned diaphragms, above which a 2½-in. wearing course of bituminous concrete was placed. Under live load this becomes a three-span continuous structure with continuity achieved by post-tensioning transversely at the ends of the cantilever. All the members are post-tensioned concrete monoliths. As might be expected, their successful construction called for many engineering and construction "firsts," developed jointly by the owner, consulting engineer, and the contractor.

The cantilever girders for the twin structures are of two types (Fig. 1). The two outer girders for each bridge are rectangular in section with nine, 9-in. diameter Sonovoids in the webs, while the four inside ones are modified I's.

Before the first girder was cast in May, 1959, the contractor spent a full year in work-planning, engineering, and resolving problems dealing with concreting and handling operations.

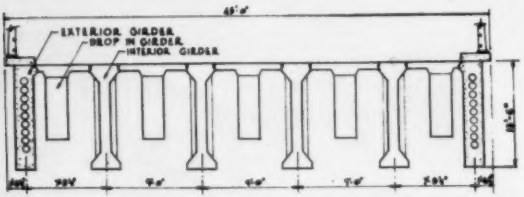

Fig. 1—Cross section of one of the twin structures for the bridge

Precasting operations

Five sets of forms were prefabricated in Syracuse, N. Y., during the winter of 1958 and consisted of: one set for the exterior girders (in 14 parts); two sets for the interior cantilevering girders (in 28 parts); two sets for the drop-in girders (in 44 parts). The forms were designed for rapid reuse as each set cost over $10,000.

There were three casting yards for the girders: the cantilevers were cast on each shore to the rear of the abutments and the drop-in girders were cast on the east side of the north bank. The casting beds were designed so that the 240 ton cantilever girders could be moved across with ease out over the abutment and pier (Fig. 2).

Concrete was delivered to the point of placement by transit-mix trucks. A unique technique of placing the concrete utilized 14 towers which supported hoppers, to which elephant trunks were attached, feeding into openings in the girder's sides. One crane using three 1-cu yd, laydown buckets alternately filled the hoppers. Concrete was fed through elephant trunks and into ports on one side of the forms and by bucket at the top of the forms. Vibrators placed through ports on the opposite side assured full consolidation between the closely spaced steel. Openings in the forms were necessary as specifications limited the height of concrete drop to 4½ ft and the large amount of steel and tendons limited placement from the top.

Work was carried out on both sides of the canal, producing one cantilever girder every 14½ days with each set of forms. It took 3 days to position the forms and to "make ready" for casting, 4 days for placing the tendons and reinforcing steel, 1 day to cast the girders, 4 days for the concrete to reach 4000 psi, 1 day for tensioning eight tendons, and 1½ days to move

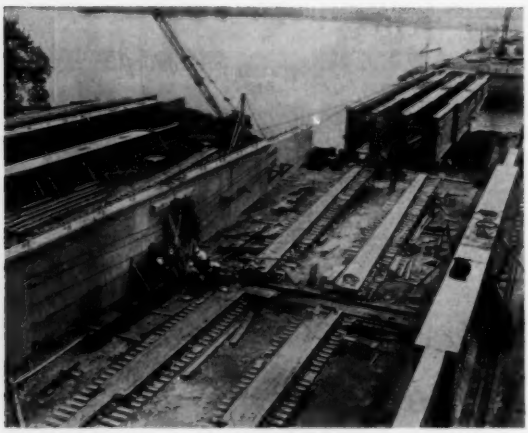

Fig. 2—At the extreme right of photo, preparations have been completed to move a 240-ton interior girder out over the water, alongside the four positioned ones. Later, the exterior girder at the left center was positioned, completing the six cantilever girders for one bridge on this side of the canal. Four 1-in. steel cables were used to stiffen the middle portion of the girder which was limber despite its massive 2 ft wide x 13½ ft section

the girder out over the water. There are 20 tendons in the exterior cantilever girders and 28 in the interior ones. Before moving, eight of the tendons were tensioned in each. Later, they were grouted from one end, using a mix consisting of three parts cement and one part fly ash with 1 lb of Intraplast B for every 2 cu ft of grout. The Intraplast made the mortar cohesive and afforded expansion to assure full encasement of the 0.25-in. diameter wires, which range in number from 24 to 28, in each 2⅛-in. conduit. Part of the counterweight and some of the diaphragms were cast and then the second stage of tensioning carried out. Diaphragms were transversely tensioned with tendons containing fourteen, 0.25-in. diameter wires. The final stage of tensioning took place after the drop-in girders had been lifted into place and then the remainder of the concrete counterweight was installed.

After partial post-tensioning in the yard, the cantilever girders were raised up on jacks at two points along their length and moved transversely on German express rollers to a position where they could be moved out over the abutment and pier which was connected by a temporary steel bridge. The cantilever was then supported on sand jacks while workmen placed the shoes and rockers.

While moving, girders were guyed with 1-in. steel cables to prevent tipping, as the heat from the sun caused the girder ends to bend as much as 2-in. Some of the tendons were tensioned from one end and some from both ends.

The drop-in girders were cast and after placement on a 40 x 200-ft car float, they were moved beneath the bridge and lifted into place between the cantilever girders. Transverse tensioning then tied together the entire system which had been partially counterweighted. All of the 12 tendons in each drop-in girder were tensioned in one phase before the 222-ton girder was moved. The 231 ft long T-girders are between 8 and 8½ ft high and 8 ft across their tops.

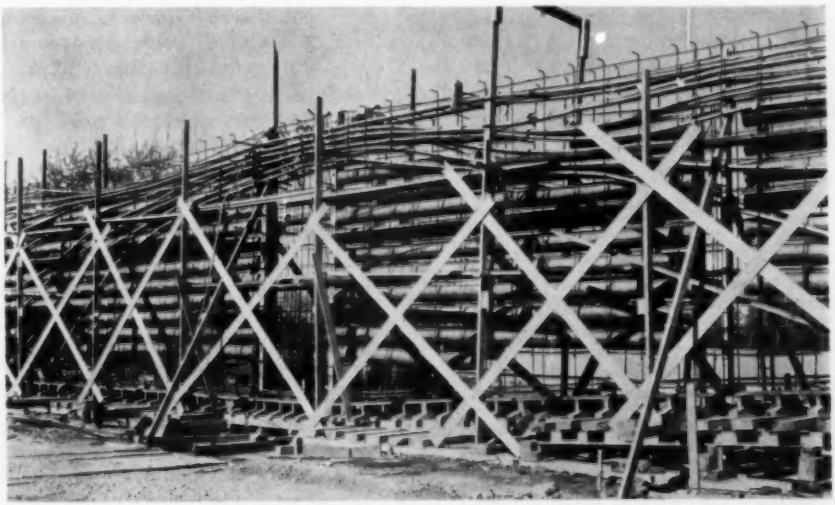

Fig. 3—In the exterior girder shown here, fiber tubes and prestressing tendons were positioned in the web before the prefabricated forms were placed.

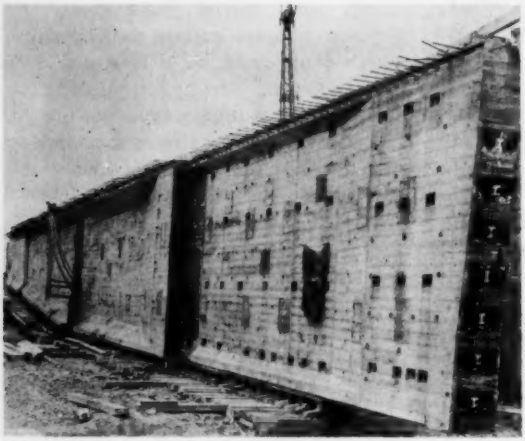

Fig. 4—The 146 x 13½ ft interior girder shown here was concreted in 5½ hr. To keep the 110-cu yd mass of concrete plastic long enough so that the entire member could be cast without cold joints, retarding densifier was added to the concrete. This was especially important as the majority of concreting was done during the summer, often in 90 deg temperatures. Location of ports for concreting the girder can be seen by the dark checkerboard pattern along the girders side.

Concrete Mix

Concrete was furnished from an automatic batching plant installed by the contractor which had a Howe scale and Batchmaster equipment. Two 1-cu yd Worthington mixers with a rated capacity of 40 cu yd per hr were used to turn out a 5-in. slump mix made of:

Concrete mix per cu yd

Cement (N. Y. State Specification Type II) (Alpha)	7¾ bags
Sand (surface dry)	1280 lb
Crushed stone (#4 to ¾ in)	900 lb
Crushed stone (⅜ in. to 1 in.)	900 lb
Total water	36 gal
Retarding densifier (Plastiment) Used in varying quantities, as influenced by temperature to maintain a slump at 5 in.	

Through the use of a retarding densifier in the mix many problems were resolved, such as:

(1) Keeping the concrete plastic until each 110-cu yd cantilever girder was cast, thereby preventing cold joints.

(2) Obtaining uniformity in the concrete as the air temperature changed by varying the proportion of the retarding densifier.

(3) Achieving good workability of the concrete without using excessive water, and thereby assuring complete filling of the small voids between the heavy reinforcing and the tendons.

(4) Accelerating the strength gain, essential to the scheduled production of two girders weekly.

Credits

The bridge was built under the direction of C. F. Blanchard, deputy chief engineer of bridges, Department of Public Works, State of New York. Summers, Munninger & Molke, Albany, N. Y., were the consulting engineers with E. Molke, partner, as project engineer. Terry Contracting, Inc., Long Island City, N. Y., was the contractor, with William Mayhew, vice-president and chief engineer, Philip A. Wolf as chief estimator and Lloyd Monroe as construction superintendent on the job.

Give
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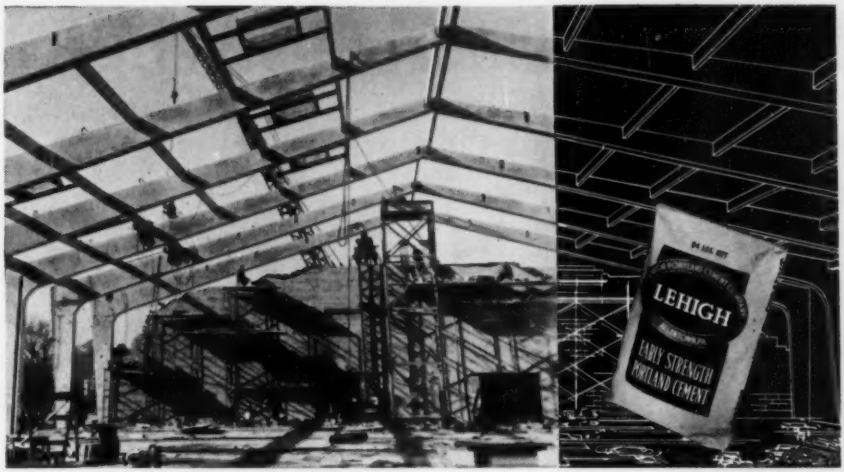


Jordan Sellars High School Gymnasium, Burlington, N. C. Unobstructed floor area is 100' by 120'. Height at center of arch is 32'.

As one crane raised and held the 18-ton arch half, another (on the right) guided it into position. The two halves were then welded together to form complete arch.

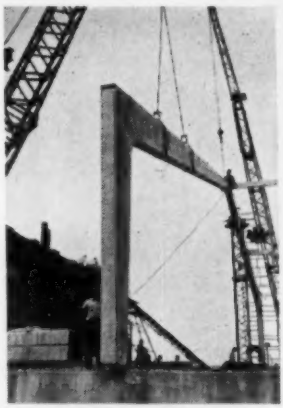
8 PRECAST CONCRETE ARCHES FRAME 100' WIDE GYM

Lehigh Early Strength Cement Cuts Casting Time

Attractiveness, durability, fire safety and unusual simplicity—all characterize the frame of this new gym and further prove the versatility of precast concrete. The huge arches were cast on the gym floor in half sections and raised into position. Precast concrete purlins, spanning 16 feet between arches, provide lateral stability.

For fast re-use of forms in casting both arches and purlins, Arnold Stone Company used concrete made with Lehigh Early Strength Cement. To quote Mr. M. A. Arnold, "Lehigh Early Strength Cement enabled us to complete the casting operations in half the time required with regular portland cement."

This is a typical advantage of Lehigh Early Strength Cement in modern concrete construction. Lehigh Portland Cement Company, Allentown, Pa.



Architect: McMinn, Norfleet & Wicker, Greensboro, N. C.

Contractor: O. G. Thompson & Sons, Burlington, N. C.

Manufacturer Precast Units: Arnold Stone Company, Greensboro, N. C.

Ready Mixed Concrete: R. F. Kirkpatrick & Sons, Burlington, N. C.

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• LEHIGH MORTAR CEMENT • LEHIGH AIR-ENTRAINING CEMENT

LEHIGH CEMENTS

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the liquid admixture that gives you 5 key controls of concrete

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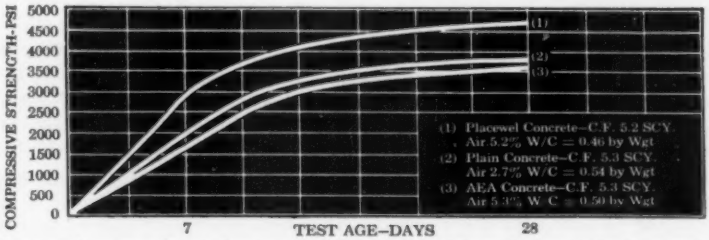
Placewel is a water-reducing, dispersing agent *plus* a catalyst. It is available with or without an air entraining agent. Placewel, by breaking up the cement flocs, releases water for lubrication that isn't normally available. Thus, mixing water requirements are cut without relying on air entrainment alone. The Placewel catalyst enhances the strength-gaining characteristics of portland cement. This unique combination of properties provides a 30% increase in concrete strength (see chart).



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Less than three ounces of Placewel per sack of cement will also –

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- Increase workability
- Improve dimensional stability
- Reduce permeability

J-M Retardwel® is recommended when concreting conditions require longer placing and finishing time. Retardwel delays the initial set of concrete and yet provides higher 24-hour strength.

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JM PRODUCTS JOHNS-MANVILLE *Celite Division*

A BIG Haul for a BIG Dam

RISING from bedrock to a height of 700 ft in the spring of 1963 will be the Glen Canyon Dam in Arizona, a monument to the men who designed it, and to the engineers, contractors, and laborers who build it.

What does it take to construct this project of such magnitude? It takes time, materials, equipment, ingenuity, personnel, and money. Who will do the job? The U. S. Bureau of Reclamation in cooperation with hundreds of contractors and thousands of workers.

The 3 year long cement haul from Phoenix Cement Co.'s Clarkdale Plant to Glen Canyon Dam will be the largest truck-ton-mile cement haul in history. The first truckload rolled up to the dam in February, 1960.

Over the 3 years 3,100,000 bbl of bulk cement will be carried in hopper trailers at total of approximately 4,320,000 miles for a total of 137,052,-000 cargo-ton-miles. Counting the trips up and back the trucks will cover a total of 8,640,00 miles.

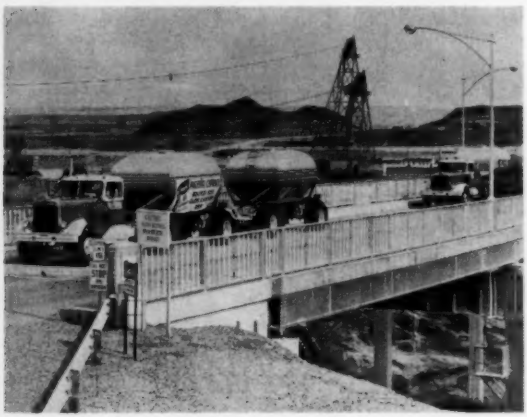
Construction of the Glen Canyon Unit of the Upper Colorado River Storage Project started on July 1, 1956, in one of the most isolated areas in the nation—the Navajo Indian Reservation in Arizona. First major jobs completed in the $421 million project were the main access road from Highway 89 (26 miles away), and the $4 million steel-arch bridge across the mammoth gorge.

The town of Page (honoring the first reclamation bureau commissioner) was planned and constructed to accommodate the hundreds of workers and their families; homes, churches, stores, recreation.

Merritt-Chapman & Scott Corp., world-wide builders, stepped into the picture on Apr. 29, 1957. M-C&S was awarded the prime contract of $107,-955,522—the largest single construction contract in the history of the USBR.

The contract called for diversion of the river and dam construction, power plant structure, and switchyard area.

Completion deadline for M-C&S was set for Mar. 4, 1964, and despite the 169-day strike which halted all operations from July 6 to Dec. 22, 1959, the over-all job is a year ahead of schedule.



First cement trucks arrive at damsite. Here they cross the $4,650,000 Glen Canyon Bridge which spans Glen Canyon just west of the damsite. Towers in background hold cables which cross canyon and are used to carry large pieces of equipment, concrete buckets, etc., from one canyon wall to the other.

Big climb through echo cut is longest climb on route from Phoenix Cement Co.'s Clarkdale plant to Glen Canyon Dam. Climb is 4.2 miles long up a 6-percent grade. This picture, taken from right side of cab of first truck and looking into right side mirror shows second section pulling the big hill. Mirror reverse image makes truck appear to be traveling on left side of road. By pretending that you are sitting in the right hand seat and taking the picture you can orient yourself.

Involved in the giant operation of producing and shipping cement to the damsite are Phoenix Cement Co., division of American Cement Corp., and Belyea Trucking Co., of Los Angeles.

After almost 20 months of construction by Fisher Contracting Co. of Phoenix, the Phoenix Cement Co. plant, with a price tag of $16 million, was ready for production on Oct. 30, 1959, at Clarkdale.

American Cement Corp. won the Bureau of Reclamation contract for providing the 3,000,000 bbl of cement by issuing low bid of approximately $3.25 a bbl, or $9,740,000. The entire production will come from the Clarkdale plant, which is capable of 5500 bbl of cement per day.

Belyea's trucking firm took on the mammoth job of transporting the cement to Glen Canyon—188 miles one way. After receiving the contract ($3 million plus) to haul the cement, Belyea built a new terminal, complete with full shop facilities, at nearby Cottonwood.

The Belyea fleet of 20 rigs, each worth $35,000 consisting of an Autocar tractor and two custom built cement trailers made by Fruehauf Trailer Co. They are powered by 220-hp Cummins diesels. Total load is 76,800 lb, including a 27¾-ton payload of cement.

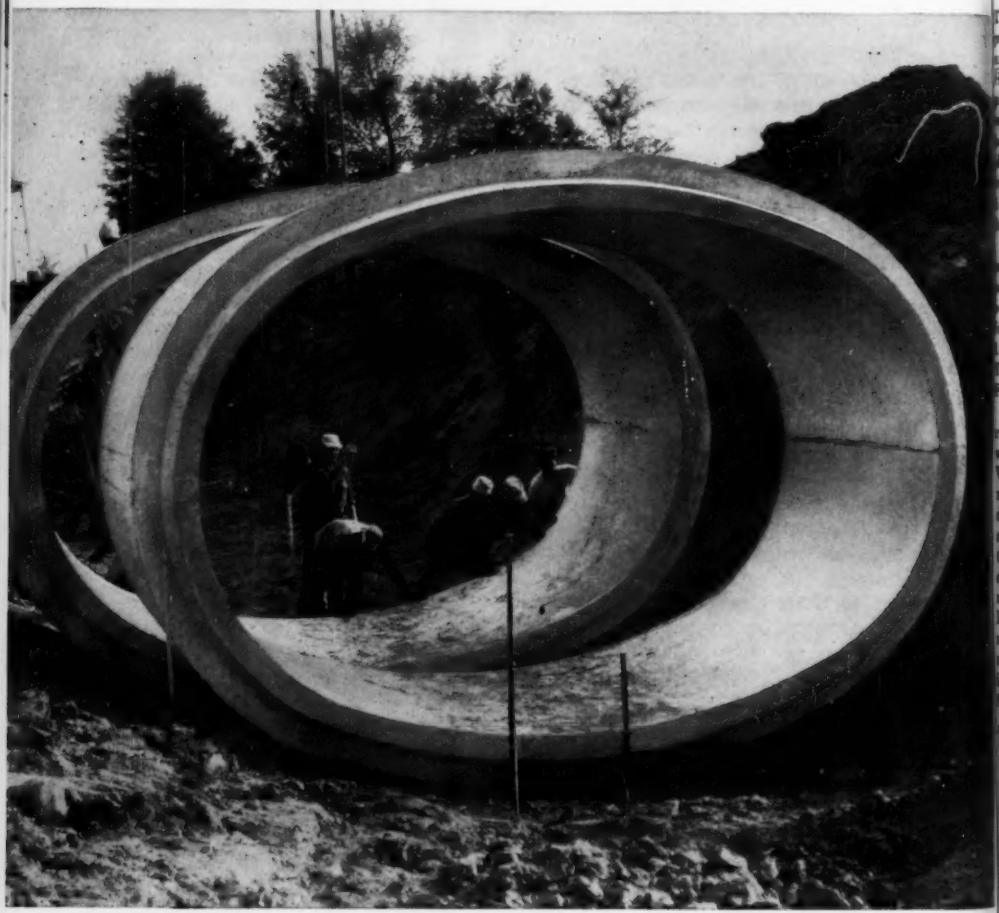
Phoenix Cement Co. and Belyea operated in the early months of the Glen Canyon contract on 980 bbl delivery per day. The maximum daily requirement will be 6000 bbl per day. The first delivery of cement in February went into concrete for lining the 50-ft diameter spillway tunnels.

Before the load is rolled to the weighing station platform, a sample is cored from each hopper by USBR chemists and tested. On acceptance of the load, it is weighed and the 27¾ tons are deposited in storage silos adjacent to the batching plant. The fully automatic batching plant is 217 ft high and has a maximum capacity of 480 cu yd per hr. The concrete will be placed with two 50-ton cableways.

All told, the dam will need 4,830,000 cu yd of concrete. Aggregate for the concrete is being produced at a plant 6 miles north of the damsite. It is transported to the batching plant by conventional bottom-dump trucks.

Other Glen Canyon projects statistics are as follows — diversion tunnels: length of right tunnel, 2749 ft, length of left tunnel, 3011 ft, diameter of tunnels, 41 ft; dam: maximum thickness, 340 ft; thickness at crest, 25 ft; pozzolan used in concrete, 220,000 tons; reinforcing steel, 10,000 tons; length at crest, 1550 ft.

USS American Welded Wire Fabric selected t



This is a 6-foot section of 144" elliptical concrete pipe. It has a greater flow capacity than its equivalent round pipe and it can be installed in a minimum depth of cut with increased depth of cover. Elliptical pipe saves headroom—allows sufficient cover to reduce frost heave.

USS American Welded Wire Fabric conforms perfectly to the elliptical shape of the 144" pipe. Because of the machine prefabricated accuracy of USS American Welded Wire Fabric, cages can be formed faster, and the spacing and concentricity of cages can be accurately controlled.

d to strengthen concrete pipe on Michigan Highway job

The Michigan State Highway Engineers faced a problem when it came to the selection of pipe for an important new highway in Berrien County. Hydraulic and grade line considerations dictated pipe with maximum water-carrying capacity with a low flow line, but with up to 33 feet of backfill. They selected over 1,000 linear feet of elliptical, reinforced concrete pipe. Diameters varied from the smallest to the largest—18" to 144".* Lamar Pipe & Tile Division, American-Marietta Company, manufactured the pipe.

In the case of the 480 feet of 144" required to withstand 33 feet of backfill, the specifications required 3 lines of reinforcement—an inner and outer cage each having 0.754 square inches per foot, and an elliptical cage having an area of 1.508 square inches per foot. Lamar Pipe & Tile elected to use American Welded Wire Fabric on this big job.

Concrete pipe manufacturers insist on quality reinforcing, meeting rigid specifications—that's why so many of them use USS American Welded Wire Fabric. This quality product—with its machine-made accuracy, assures the proper distribution of steel because the wire diameters are held to the close tolerance of ±0.003" and their spacing may not vary by more than ¼" center-to-center. This prefabricated product is more accurate than other forms of reinforcing. Its cold-drawn, high-tensile steel wires have a minimum yield point of 60,000 psi and a minimum ultimate strength of 75,000 psi. For more information, write to American Steel & Wire, 614 Superior Avenue, N.W., Cleveland 13, Ohio.

*Round equivalent

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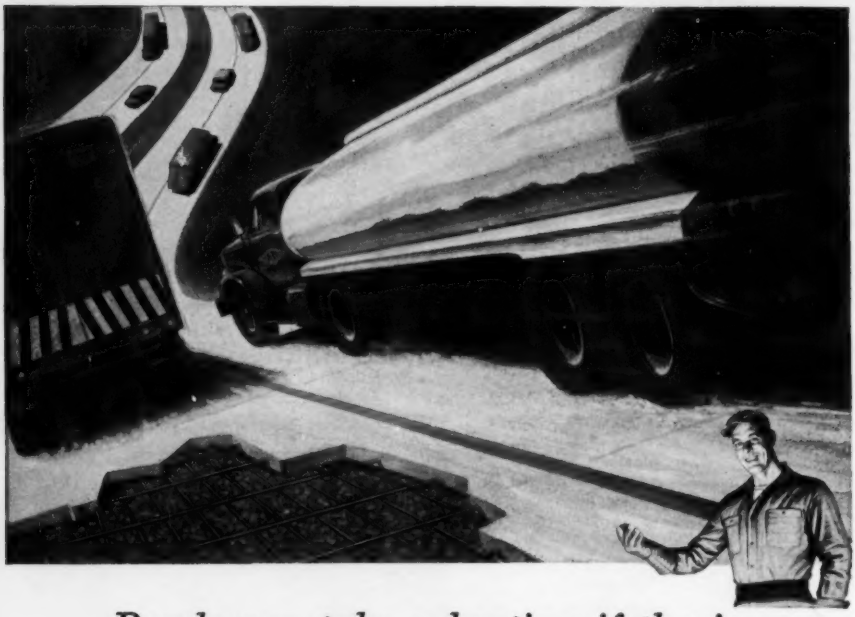
American Steel & Wire Division of United States Steel

Columbia-Geneva Steel Division, San Francisco, Pacific Coast Distributors
Tennessee Coal & Iron Division, Fairfield, Ala., Southern Distributors
United States Steel Export Company, Distributors Abroad

To increase the strength of the pipe, by resisting diagonal tension, 320 ¾" diameter stirrups are attached through the three cages of USS American Welded Wire Fabric.



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If you're trying to build better, longer-lasting concrete highways, contact your nearest CF&I sales office before starting your next job. Our engineering staff will be happy to discuss your reinforcing requirements with you. That's one of the meanings of our Corporate Image — the best of service to the concrete industry.

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Positions and Projects

Three USBR engineers abroad on European dam study

Three Bureau of Reclamation engineers on the staff of Assistant Commissioner and Chief Engineer Grant Bloodgood, at the Denver Federal Center, are abroad for a 2-month study of European dams.

The three men are: Merlin D. Copen, George C. Rouse, and ACI member George B. Wallace.

As a result of their study, the engineers expect to obtain information on dam design techniques that will be a basis for comparison between United States and European practice. The current European trend toward thin-arch dams will receive attention by the Bureau engineers who will compare the safety factors of such structures with those of the thicker dams built in this country. They will inspect dams in Portugal, Italy, Switzerland, and France, and discuss design, construction, and research problems with leading European authorities. Among the structures they plan to visit is the 870 ft high Vaiont Dam, the world's highest arch dam, now nearing completion on the Vaiont River in northern Italy.

Each of the Bureau engineers is a specialist. Mr. Copen is an expert in the mathematical analyses of the various stresses in concrete dams. Dr. Rouse specializes in structural model studies and in instrumentation for mass concrete dams. Such instruments provide much of the engineering data used by designers in determining the effect of water loads on dams and their foundations, stresses caused by temperature changes, effects of earthquakes, and other factors. Mr. Wallace is concerned with construction methods and materials, including the investigation of the engineering properties of cements, aggregates, and other materials used in the manufacture of concrete.

Amirikian awarded degree in Vienna

Arsham Amirikian, special structures consultant, Bureau of Yards and Docks, Department of Navy, Washington, D.C., recently received the degree of DSc from the Technical Institute of Vienna. The Institute is one of the oldest engineering schools in Europe, established in 1815. His thesis for his doctorate work dealt with the analysis and design of structures subjected to nuclear blast.

Dr. Amirikian is a member of the Institute's Board of Direction, chairman of ACI Committee 324, Precast Reinforced Concrete, Thin Sections, and a member of Committees 116, Nomenclature, and 314, Rigid Frames for Buildings and Bridges.

He has been widely honored for his outstanding contributions to the development of reinforced concrete. In 1958 he was awarded the ACI Alfred E. Lindau Award in recognition of his contributions to precast reinforced concrete construction.

Maxwell Upson retires with 53 years service

Maxwell M. Upson, distinguished foundation and harbor engineer, recently retired after 53 years continuous service with Raymond International, Inc., New York. He will continue to serve as a director and honorary chairman of the board.

Mr. Upson joined the company in 1907 as secretary, general manager, and chief engineer. In 1930 he was named president and since 1946 has been chairman of the board. Under his tenure as chief executive officer and due in great part to his vision, pioneering and "daring to do," Raymond has skyrocketed into a multimillion dollar concern with approximately 60 subsidiaries, and more than 7000 Americans and an additional several thousand foreign nationals on its payroll.

It has some 50 offices scattered around the world, and can point to almost 42,000 completed contracts.

Mr. Upson pioneered the use of reinforced concrete, the use of prestressed concrete, and his hollow-core prestressed piles are today an important source of revenue for the company. More than 50 patents on a variety of inventions now stand to his credit.

He joined ACI in 1921 and was president of the Institute in 1926. He has been a member of ACI's Board of Direction, active in technical committee work, and a contributor of technical papers for JOURNAL publication.

Harrenstien joins faculty at University of Arizona

Howard P. Harrenstien, formerly faculty member at Iowa State University, Ames, has joined the staff at University of Arizona, Tucson, as associate professor of civil engineering.

Permanente Cement announces plans for Alaska plant

Permanente Cement Co., Oakland, Calif., has filed claims covering 240 acres of high-grade limestone deposits near Anchorage, Alaska. The limestone deposits will serve as sources of raw material for a cement manufacturing plant which Permanente plans to build to serve Alaska's growing construction market.

A site has been acquired at Sutton, adjacent to the Alaska Railroad line, where the plant will be located. The most recent engineering advances will be incorporated in the facilities, which will represent an investment of approximately $5 million. Initial productive capacity will be approximately 500,000 bbl of cement per year with provision for future expansion. The company's existing distribution facilities at Anchorage and Fairbanks will continue to function as key distributing points. Other distribution depots may be added as needed to meet market requirements, either for ship, barge, rail, or truck distribution.

Amirikian attends Stockholm precast concrete congress

Arsham Amirikian, special structures consultant, Bureau of Yards and Docks, Department of Navy, Washington, D.C., represented ACI at the International Congress of the Precast Concrete Industry, June 16-24 in Stockholm, Sweden. This is the third of triannual meetings sponsored by a group of concrete products manufacturers' associations representing some ten western European countries.

The first congress took place in Brussels, Belgium, in 1954, and the second one in Wiesbaden, Germany, 1957. These meeetings serve as international forum for the discussion of problems and the presentation of the latest developments in the field of precast concrete construction. In addition to the technical sessions, an industrial show is also arranged for the display of the products and machinery of the precast concrete industry.

The technical program of the Stockholm congress covered 3 all-day meetings, featured by general reports, group discussions and brief presentations. The general themes included the production of large precast concrete walls for buildings, the shrinkage of concrete, and the practical applications of vibration to the manufacture of precast concrete products. Concerning special problems, there were such topics as the protection of concrete pipes against the attack of aggressive soils and waters by the use of plastic coverings, the electro-curing of concrete products, dimensional tolerances in prefabrication, as well as other research and production studies associated with precast concrete.

The exhibit consisted of both outdoor and indoor displays and demonstrations. The outdoor show was featured by a series of structures whose components, produced by the Swedish precast concrete manufacturers, were brought to the site and erected for demonstration. These fram-

ing components covered a wide range of elements, varying from lightweight planks for house framings, to huge wall panels, tristory column shafts and large-span girders for industrial buildings. Also of special interest were such other castings as spiral staircase segments, ornamental panels, statuary pieces, and prestressed railroad ties. In the indoor exhibit, the main attraction was an impressive photographic assembly illustrating important examples of European precast concrete construction. This display was supplemented by a series of motion picture films, some of them with English narrations.

Over 900 registrants from some 22 countries, including 31 from the United States, attended the convention. In view of the fact that the United States is not as yet affiliated with the governing body of the congress, Bureau International du Béton Manufacture, better known by its initials, BIBM, this number of individual participants may be considered as quite significant.

As an adjunct to the technical activities, the program included the customary social events, local inspection trips, and sightseeing tours. For those desiring more extended journeys through the country, week-long pre- and post-congress tours were also arranged.

Houston Contracting acquires Rackle Co.

Houston Contracting Co., Houston, one of the largest builders of cross-country pipelines for the oil and gas industry, has acquired the Rackle Co. of Texas as a wholly-owned subsidiary from the George Rackle & Sons Co., Cleveland, Ohio.

John C. Fitch, vice-president and general manager of Rackle of Texas under its former owners is president of the new subsidiary. Officers in addition to Mr. Fitch, will be R. P. Gregory, president of Houston Contracting, who will act as vice-president and Albert W. Michael, secretary-treasurer.

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Rüsch awarded honorary degree

Hubert Rüsch, professor and director of the Engineering Materials Laboratory, Technical University, Munich, Germany, has been awarded the honorary title of Doctor-Engineer by the Technischen Hochschule, Dresden, Germany.

Dr. Rüsch has won much prominence in Europe and South America through his design and construction of outstanding reinforced concrete structures in Europe, England, and South America. He has played an active part in the development of shell structures, prestressed concrete, and precast construction.

Dr. Rüsch is currently a member of ACI Committee 115, Research.

Larson retires

Guy H. Larson has announced his retirement from the Wisconsin State Highway Commission.

Anderson joins Texas firm

F. David Anderson, formerly associated with Stone and Webster Construction Co. in Sao Paulo, Brazil, has joined Vowell Construction Co., El Paso, Tex.

PCA awarded National Safety Council citation

Portland Cement Association, Chicago, was named a winner of the National Safety Council's Association award for 1960.

PCA was one of ten associations, representing various industries in the United States and Canada, cited by the Council for excellence and balance in their safety programs and above-average progress in accident prevention.

The ten winning associations represent 2447 firms employing 2,272,225 persons.

Barreto joins Rogers firm

Renato G. Barreto has been appointed an associate of Paul Rogers and Associates, Inc., consulting engineering firm, Chicago. Mr. Barreto obtained his MS degree in civil engineering at the University of Notre Dame. A fluent linguist, he is expected to handle much of the firm's contemplated European projects.

Proposed ASTM cement specification

A proposed specification for additions used in processing portland cement was accepted by ASTM Committee C-1 on Cement during the ASTM Annual Meeting at Atlantic City, N. J., June 26 to July 1, 1960. The specification was completed after a long and intensive study by a subcommittee. The new specification will relieve the committee of the responsibility of approving additions and will place this responsibility in the hands of the manufacturer who would be guided by the specification. The action is subject to ratification by letter ballot of the committee and approval by the ASTM Administrative Committee on Standards sometime this fall.

New methods of tests were approved for determining the amount of water-soluble alkali in masonry cements as well as for evaluating false set and potential sulfate resistance of portland cement. The chemical requirements for portland cement in portland blast-furnace slag cement will contain several changes as recommended to the committee. An increase in the autoclave specimens limited for portland cement is also under consideration. A cooperative series of tests to determine the influence of time of wait during mixing of mortars in the testing of masonry cement has been completed with results obtained from 21 laboratories. This test program is part of a joint study by both Committee C-1 on Cement and C-12 on Masonry Mortars.

ECONOMICAL

Job illustrated: St. Louis Avenue Underpass, Mark Twain Expressway, St. Louis, Mo., Contractor: Fred Weber Contractor, Inc., Designer: Missouri State Highway Department, Rex M. Whitton, Chief Highway Engineer; J. A. Williams, Bridge Engineer, Consulting Engineers: Sverdrup & Parcel Engineering Co.

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Only Addex Roof Shield TS-2 has proved itself capable of meeting so many thin shell roofing requirements with consistently excellent performance.

Result: Addex thin shell roofing specifications and materials have been endorsed by architects, engineers and roofers.

Addex Roof Shield Specification TS-2 calls for (a) priming on deck areas and deck edge faces, (b) glass fiber reinforced waterproofing over open deck areas and (c) decorative, reflective white surfacing with performance characteristics to match the waterproofing.

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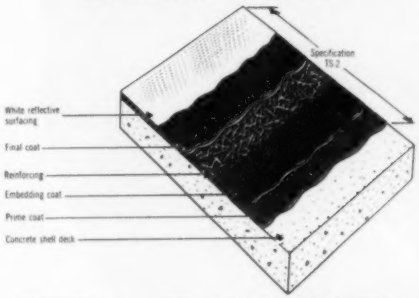

Prime coat . . . . Addex No. 480 Asphalt Emulsion thinned with 30% water
Embedding coat . . . . . Addex No. 480 Asphalt Emulsion
Reinforcing . . . . . Addex Heavy Duty Glass Fiber Mesh
Final coat . . . . . Addex No. 480 Asphalt Emulsion
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Advantage: uniformly high tensile strength in every direction.

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3 — a pure white roof surfacing emulsion containing a light resin binder to insure a firm bond to asphalt and create a highly reflective, decorative, weatherproof, self-cleaning surface.

Advantages: beauty, durability, weather resistance.

Addex materials have been thoroughly field tested. All have outstanding performance records and current wide usage among roofers and roofing contractors as components for weatherproofing specifications on masonry surfaces and on smooth asphalt roofs.

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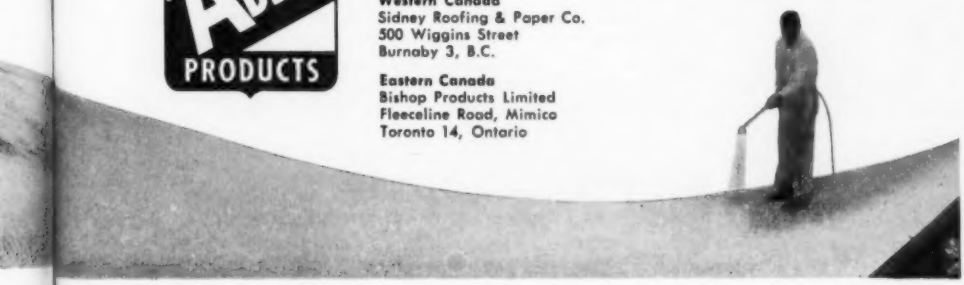

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FOR HUMBER SEWAGE TREATMENT PLANT

Contractor: Dravo of Canada, Ltd., Toronto.
Engineers: James MacLaren & Associates, Toronto.

It's king-size for Canada, but commonplace in the United States—this 16-foot section of 10-foot diameter precast concrete pipe.

Joined by "Pressure Proven" rubber and steel AMSEAL Joints to provide a positive, water-tight seal against infiltration or leakage, over 2000 feet of the 120" pipe is used in the Toronto subacqueous outfall sewer. Other sections include lengths of 108", 96", 84", 72" and 54" diameter pipe.

American-Marietta's up-to-date facilities, strategically located throughout the country, assure prompt and regular deliveries. Long lengths speed installation, too, by reducing handling and joint making.

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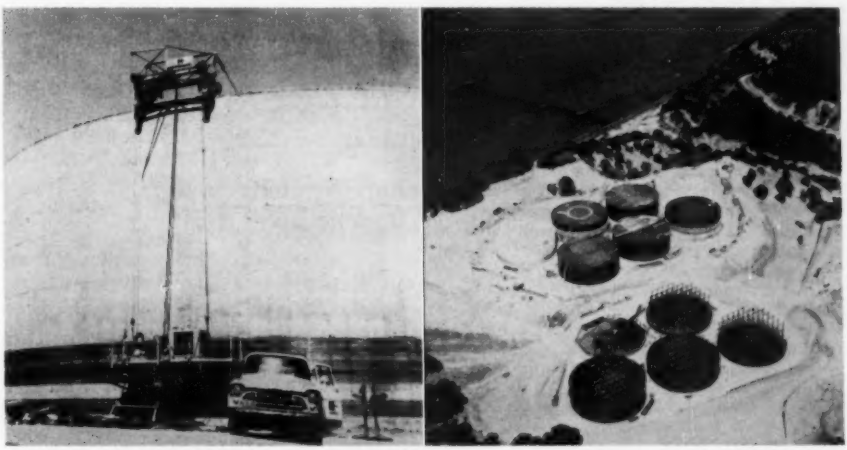
Underground Prestressed Jet Fuel Tanks Designed to Withstand Atomic Blasts

TWELVE UNDERGROUND, steel-lined, prestressed concrete tanks for storing aviation fuel for the Air Force at Ozol, Calif., were designed to withstand atomic explosions.

Each tank has a 15 in. thick floor slab, 9 in. thick prestressed concrete wall, and 9 in. thick concrete roof. Each is continuously lined with ¼-in. steel plates welded together. The walls are prestressed to withstand outward forces which will be exerted by tank contents and exterior loadings. Prestressing consisted of wrapping the tank with layers of prestressed wire. Each of the tanks has an inside diameter of 122 ft 3 in., a clear height of 40 ft, and a storage capacity of 83,500 bbl.

Prior to placing the floor slab, angles were set in place with the horizontal leg at floor level and the vertical leg extending down into the space to be concreted. The floor plates, delivered to the site in sections 30 to 40 ft long by 8 ft wide, were later welded to these angles and all joints tested for leakproofness.

Roof-supporting columns were next installed, after which the preshaped wall liner was erected, welded in place, and tested for leakproofness. The joint where the wall liner meets the floor liner was welded all around and a backup angle installed in the outside corner. This construction restrains horizontal movement of the wall at its base, an important consideration



Left—Wire-winding machine suspended from carriage winds wire around tank at speed of up to 8 miles per hr. Right—Steel-lined, prestressed-concrete fuel-tank farm for aviation fuel showing tanks at various stages of completion

in case of blast loading. At the same time, it safely allows for wall rotation at the joint, while maintaining a leakproof connection. Anchor-studs were welded to the outside of the erected liner to secure bond between the liner to the subsequently placed concrete wall.

The concrete wall was placed in successive rings, the steel liner acting as an inner form. Outer forms were held in position with special tie-clamps attached to the anchor studs.

Roof plates were next welded in position and the roof slab placed, with provisions for manways, vents, and gage hatches. The joint between roof and wall is similar to the joint between floor and wall.

The prestressing wire was then wound around the tank, being drawn in the process so that it was applied under tension of 150,000 psi. The wire-winding machine was suspended along the outside wall from a carriage which ran along the outer edge of the roof. This carriage was securely guyed by radial cables to the center of the roof. The compressive stress imparted to the tank wall by the wires varied from 234 kips per vertical ft at the bottom of the tank to 34.5 kips per vertical ft at the top.

A ¾-in. coat of pneumatic mortar was applied to protect and bond the wire and this in turn was covered with a waterproof coating. After completion of the tanks, the site was graded to provide a 4-ft covering over each tank.

The tanks are owned by Lark Corp., Dallas, Tex., and have been leased to the Military Petroleum Supply Agency. The prime construction contract was held by Herrick Iron Works, Hayward, Calif., who furnished reinforcing and structural steel as well as the prestressed reinforcement; prestressing was by the Herrick Iron Works under a license from The Preload Co., Inc. Concrete work was done by Ben C. Gerwick, Inc., steel plate work was done by Yuba Industries and Pittsburgh Des Moines Steel Co.

Concrete Pavements and Bases

SPECIFICATIONS FOR CONCRETE PAVEMENTS AND CONCRETE BASES (ACI 617-58). This ACI Standard covers the construction of portland cement concrete pavements and bases under normal conditions for both highways and airports. Subjects discussed include materials, dimensions, setting and removal of forms, and construction methods. Concrete proportioning based on design for minimum strength is covered in detail. Other sections cover production of high-early-strength concrete, the handling and mixing of materials, and requirements for placing, finishing and curing of pavement concrete. 29 pp. Price $1.00, 50c to ACI members.

AMERICAN CONCRETE INSTITUTE PUBLICATIONS

Galla assumes Ghana post

Francis S. Galla has left the Henry J. Kaiser Co., Berkeley, Calif., engineering staff, to assume a post with their overseas operation, Kaiser Engineers and Contractors, Inc., Tema, Ghana.

Appointments to Dundee sales staff

Six sales representatives have been appointed by Dundee Cement Co., Dundee, Mich., to augment their sales staff in various district offices.

George Zuba has been appointed to the Chicago district office; E. Douglas Holwadel, Cincinnati area; and Robert W. Taylor, Ohio district office with headquarters in Cleveland. Earl A. Barker, John J. Leavy, Jr., and Jack R. Robinson have been appointed to the Detroit district office.

UNIVERSITY OF HARTFORD General Classroom Building, costing more than $2 million including site work, is the first completed structure in the initial phase of a campus development program on 150 acres in West Hartford.

The structural frame is reinforced concrete throughout except the lecture hall which is steel. The building consists of a two-story wing and a four-story wing in a modified T shape. Floor area of 69,000 sq ft provides more than 125 class, seminar, conference, and lecture rooms, laboratories and offices.

In addition to the building construction of a central heating plant has been completed. It has been designed for ready expansion as new buildings are added.

Roads have been built, parking areas paved, an underground sanitary system installed and a beginning made on an underground installation for all utilities.

The first phase of the development program calls for the construction of eight buildings designated: Engineering and Science Building, Auditorium and Music Buildings, Business Administration Building, Students' Center and Art School Building, including a sculpture court.

Moore and Salsbury, West Hartford, were architects on the classroom building and A. F. Peaslee, Inc., Hartford, general contractor.

Model home sparks new interest in concrete block

The New England Concrete Masonry Association in cooperation with *Good Housekeeping* magazine has completed a model home project which shows that concrete masonry is not only suited for contemporary design. The home, constructed entirely of concrete block, illustrates its effectiveness in traditional architecture as well.

A Cape Cod style, the home is located in Lynnfield, Mass., a new suburban development north of Boston, consisting entirely of custom-built homes. Virtually every type of concrete masonry available was used inside and outside the house, running a gamut from the cinder block exterior faced with split-block to colored glazed block in a modern utility room.

Designed to meet the increasing demand for traditional architecture in residential building, the NECMA-*Good Housekeeping* model home predicts a prominent role for concrete masonry in the rapidly expanding new home market during the next decade.

PCA announces promotions in Michigan district

Promotion of three Portland Cement Association engineers and reassignment of duties in Michigan has been announced by J. Gardner Martin, Michigan district engineer for PCA.

T. M. Reynolds, Lansing, was named field engineer supervisor and E. H. Wiley, Okemos, was appointed state paving engineer. W. C. Krell, Detroit, was named supervising engineer in the structural and housing fields.

ACI member firms engaged in new United Engineering Center

Two ACI member firms are engaged in the erection of the tower of glass, stainless steel, and limestone that will house the United Engineering Center on United Nations Plaza in New York. Turner Construction Co., New York, is general contractor; structural engineering is being supervised by Seelye, Stevenson, Value and Knecht, New York.

When completed, the building will be 20 stories high with 260,000 sq ft of floor space; 18 stories in office space and 2 in mechanical equipment.

Australia moves to establish prestressed concrete code

The Standards Association of Australia has set up a technical committee on prestressed concrete to produce an Australian code for prestressed concrete structures.

ACI member James M. Antill, consulting construction engineer, Sydney, is chairman of the committee, which is comprised of representatives from various Australian states. Mr. Antill was one of the pioneers of prestressed concrete construction in Australia 10 years ago.

BRI announces new officers

Harold L. Humes, Baldwin-Ehret-Hill Co., Trenton, N.J., recently began his second term as president of the Building Research Institute, and Robert W. Cutler, Skidmore, Owings and Merrill, New York, his second term as vice-president.

Newly elected vice-presidents are: Graham J. Morgan, United States Gypsum Co., Chicago, and Peter B. Gordon, Wolff and Munier, Inc., New York.

Appointed by the National Academy of Sciences-National Research Council, parent organization of the Institute, to serve 3-year terms on the BRI board of governors are: Jack E. Gaston, Armstrong Cork Co., Lancaster, Pa.; Grayson Gill, Grayson Gill, Inc., Dallas, Tex.; T. F. Olt, Armco Steel Corp., Middletown, Ohio; Howard C. Hardy, Howard C. Hardy and Associates, Chicago; and Walter Sanders, University of Michigan, Ann Arbor.

Caldwell elected V-P of American Cement

William M. Caldwell has been elected vice-president in charge of finance of American Cement Corp., Philadelphia. Mr. Caldwell will headquarter in the Los Angeles office of American Cement. Prior to joining the company, he was treasurer and controller of Drewry Photocolor Corp., Glendale, Calif.

Nicholson honored by Denison University

James A. Nicholson, president of Nicholson Concrete and Supply Co., Toledo, Ohio, was cited by Denison University, Granville, Ohio, in recognition of "outstanding achievements and services which reflect honor" on the institution. Mr. Nicholson received the PhB degree from Denison in 1927 and an LlB from the University of Toledo in 1936.

Mr. Nicholson founded Nicholson Concrete Co. and its subsidiaries, Toledo Plaster and Supply Co. and Penn Sand Co.

Hercules announces executive appointments

Herbert G. Sobelton has been appointed vice-president in charge of administration of Hercules Cement Co., Philadelphia. He has been active in various capacities throughout the company during the past 9 years.

John J. McGonagle has been appointed sales manager of Hercules. He has been associated with the company for the past 12 years and has held various positions in the sales department.

When temperatures drop, Columbia Calcium Chloride in the mix keeps concrete jobs moving.

TEMPERATURE: 48°F JOB: ON SCHEDULE

with Columbia Calcium Chloride in the mix

Faced with winter concreting and a time-tight schedule? Columbia Calcium Chloride in the mix any time the temperature drops below 70°F helps prevent lag in strength gain. Your jobs keep *moving* in spite of low temperatures. You get these specific time-and-money-saving benefits:

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FASTER INITIAL SET—Under normal conditions, Columbia Calcium Chloride reduces initial set from 3 hours to 1 hour. At lower temperatures, the accelerating effects are even more pronounced. You get finishers on and off the job faster and avoid costly overtime.

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Get all these Columbia Calcium Chloride cold weather benefits on your next job. Just add Columbia Calcium Chloride at the rate of 2% by weight of cement. Want technical help? Our calcium chloride engineers are at your service. For their assistance, just contact our District Sales Office nearest you or write to our Pittsburgh Office.

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LOOKING AHEAD

Oct. 10-14, 1960 — Annual Convention, American Society of Civil Engineers, Hotel Statler, Boston, Mass.

Oct. 13-15, 1960—75th Anniversary and National Convention, Tau Beta Pi, Lehigh University, Bethlehem, Pa.

Oct. 31-Nov. 1-2, 1960—13th Regional Meeting, American Concrete Institute, Pioneer Hotel, Tucson, Ariz.

Nov. 3-4, 1960—Annual Meeting, National Slag Association, Hotel Mayflower, Washington, D.C.

Nov. 14-15, 1960—Prestressed Concrete Conference, University of California Extension and the University of California Department of Engineering, Biltmore Hotel, Los Angeles, Calif.

Nov. 14-19, 1960 — Public Works and Municipal Services Congress and Exhibition, Olympia, London, England

Nov. 15-17—Fall Conferences, Building Research Institute, Shoreham Hotel, Washington, D.C.

Nov. 21-22, 1960—Prestressed Concrete Conference, University Extension and the University of California Department of Engineering, Sheraton-Palace Hotel, San Francisco, Calif.

Nov. 21-26, 1960—Symposium on Concrete Pavements, Buenos Aires, Argentina

Jan. 5-7, 1961 — FIP-RILEM Symposium on Injection Grout for Prestressed Concrete, Norges Tekniske Hogskole, Trondheim, Norway

Jan. 17-19, 1961—44th Annual Convention, National Crushed Stone Association, Hotel Americana, Bal Harbour, Fla.

Jan. 23-26, 1961—31st Annual Convention, National Ready Mixed Concrete Association, Hotel Americana, Bal Harbour, Fla.

Jan. 30-Feb. 2, 1961—12th Biennial Concrete Industries Exposition, National Concrete Masonry Association, Cobo Hall, Detroit, Mich.

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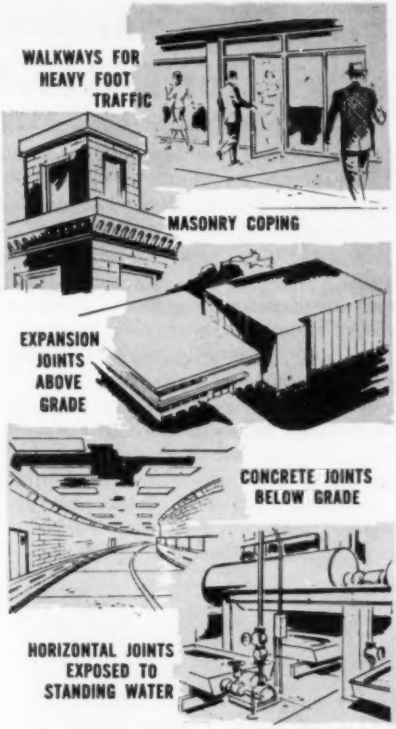
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Thin Bonded Resurfacing for Louisville Airport

The first thin-bonded concrete resurfacing project for a municipal airport in the United States was recently completed at Standiford Field, Louisville, Ky.

The relatively new technique for restoring smooth surfaces to old pavement was previously confined to several Air Force bases.

George M. Eady Co., Louisville, was contractor for placing a 4 in. thin-bonded concrete overlay on a portion of a runway at the Louisville airport.

Research by the Portland Cement Association has determined that thin layers from 1 to 4 in. thick provide adequate bond if old concrete surfaces are properly cleaned and prepared.

Steps to obtain a clean sound surface include the use of a scarifying machine to rout and chip out loose material and sealer in joints and cracks, and washing down the surface with detergent and acid baths.

After thorough flushing with water to neutralize the detergent and acid, a 1:1 cement-sand grout is brushed over the surface as a bonding course between the old slab and the resurfacing.

Eady used ready-mixed concrete on the Louisville job. Resurfacing was placed in 25-ft wide lanes across the total 150-ft width of the runway. Fiber strips were used for longitudinal joints, while transverse joints were sawed.

Ready mixed concrete was placed for thin bonded resurfacing of a portion of a runway at Standiford Field

Photos Courtesy Portland Cement Association

Fiber strips formed longitudinal joints and transverse joints were sawed

Errata

The following correction should be made in the discussion of "Evaluation of Concrete and Mortar Mixes" which appeared in the Part 2, Sept. 1960 ACI JOURNAL.

p. 1388 — in the tenth line change "0.50 ± 0.5" to "0.50 ± 0.05."

* * *

The following corrections should be made in the discussions of "Distribution of Torsion and Bending Moments in Connected Beams and Slabs" which appeared in the Part 2, Sept. 1960 ACI JOURNAL.

p. 1435 — in the first line change "Eq. (24)" to "Eq. (37)."

p. 1436—in the caption to Fig. H change "K_s" to "k_s."

p. 1437—in the two equations at the bottom of the page change "K_s to k_s."

p. 1438 — in the first equation the term "$G/E \approx 0.43$" is not part of the equation for β.

Keep on schedule all winter
—order your ready-mix

SPECIAL WINTERIZED *with* **SOLVAY CALCIUM CHLORIDE**

Ask your ready-mix supplier to include 2% Solvay® Calcium Chloride,* heated water, aggregate. Keep on schedule at *any* temperature with this "Special-Winterized" mix that . . .

Reduces overtime finishing. Sets faster. **Speeds form removal.** Develops high early strength. **Reduces protection time** up to 50%. **Reduces delay** between operations. **Adds safety** through extra cold weather protection.

Your concrete also has 8 to 12% greater ultimate strength. It's more workable. With *less* water-to-cement, you get denser concrete—more resistant to moisture and wear.

*Speeds but does not change the basic action of portland cement. This use of calcium chloride is approved by American Concrete Institute.

Write for *Solvay's 38-p. "The Effects of Calcium Chloride on Portland Cement."*

Allied Chemical

SOLVAY PROCESS DIVISION
61 Broadway, New York 6, N. Y.

SOLVAY branch offices and dealers are located in major centers from coast to coast.

p. 1438—in the eighth line from the top change "K_{cd}" to "K_{Column}."

p. 1438—in the first, third and fifth lines under "Flexural stiffness of slab panels . . ." change "K_s" to "k_s."

p. 1438—in the second line under the heading "Distribution Factors" change "K_s" to "k_s."

p. 1438—in the fourth line under the heading "Distribution Factors" change "μ" to "$1 - \mu$."

p. 1439—in each of the equations under "at A" and "at B" change $\times$ to $\sim$.

p. 1440—in the last two lines change "M_A" to "m_A."

p. 1441—in the first line above the figure change "M_o" to "m_o."

p. 1441—in the fourth line change "M_s" to "m_s."

p. 1441—in Eq. (37) change "M_o/M_A" to "m_o/m_A," and "Fig. D" to "Fig. H."

p. 1441—in the fifth line above the figure change "M_B" to "m_B."

p. 1441—change the caption to Fig. K to read "Moment diagrams for slabs and beams."

p. 1442—in the fifth line change "M_B" to "m_B."

p. 1442—in the eighth line change "M_o" to "m_o."

p. 1442—in the tenth and eleventh lines changes "ΣM_B" and "ΣM_o," respectively, to "Σm_B" and "Σm_o," respectively.

p. 1446—in the equation for J, b^2 should be b^3.

Dewey and Almy Appoints Ellis to new post

Donald T. Ellis has been appointed general manager of the Container and Chemical Specialties Division of the Dewey and Almy Chemical Division, W. R. Grace and Co., Cambridge, Mass. Mr. Ellis joined the company in 1959.

Andre Coyne

The death of Andre Coyne in Paris recently at the age of 69 ended the career of one of France's greatest engineers. His contributions to the art and science of dam design were recognized and utilized throughout the world.

During his professional lifetime he was responsible for the design of 65 major dams, most of them in France, but others in Morocco, Algeria, Rhodesia, India, and Indonesia. Many of these dams incorporated notable design advances and innovations. He was a particular advocate of concrete arch dams.

The last great Coyne-designed dam to be completed was the Kariba Gorge arch in Rhodesia, which was dedicated in May of this year. Approaching completion is Roseland Dam in the French Alps, a unique combination of the arch and buttress types. Other notable Coyne-inspired advances include the ski-jump spillway, prestressed cables to tie dams to their foundations, and the development of the domed arch.

Mr. Coyne's early career was spent in the service of the French government, but in 1947 he set up his own consulting firm of Coyne and Bellier, whose clients have come from countries on every continent.

From 1946 to 1952 Mr. Coyne was president of the International Commission for Large Dams.

David B. Steinman

David B. Steinman, internationally famous engineer, who designed more than 400 bridges in a half century, died recently at the age of 73.

In preparation for his lifework, Dr. Steinman spent 6 years at the College of the City of New York, 4 at Columbia University, earning the degrees of BS, AM, CE, and PhD, and numerous scholarships, fellowships, and awards. The following 12 years he spent in alternate teaching and practice.

In 1921, Dr. Steinman commenced his practice as a consulting engineer, specializing in the design and construction of long-span bridges.

Known and honored the world over as the designer and builder of beautiful bridges, his professional assignments have spanned the continents. Governmental agencies, both here and abroad, professional societies, and universities have bestowed countless honors on Dr. Steinman in recognition of his professional achievements and scientific contributions to the design and construction of bridges. The Mackinac Bridge connecting the two Michigan peninsulas was one of his most recent accomplishments.

He was past 60 when he turned from the drafting table to compose poetry, and his poetic works have been printed in many publications and anthologies. He believed that pure mathematics, expressed in the concrete and steel of a well-designed bridge, was in itself poetry.

Dr. Steinman had been a member of ACI since 1927.

Precast Concrete Floor, Roof Units

MINIMUM STANDARD REQUIREMENTS FOR PRECAST CONCRETE FLOOR AND ROOF UNITS (ACI 711-58). This ACI Standard presents requirements for single units or multiple element assemblies, to be used in conjunction with ACI 318-56. Covers materials, design principles, manufacture including curing and handling, testing of completed units, installation plans, and special provisions for holes and openings in members. Design chapter discusses dimensions, allowable deflection, structural concrete topping, reinforcement anchorage and location, and use of lightweight concrete. 12 pp. Price 50c, 40c to ACI members.

AMERICAN CONCRETE INSTITUTE PUBLICATIONS

Membership in the American Concrete Institute

To facilitate prospective members in joining the Institute, membership application forms are provided. Present Members may aid by bringing these forms to the attention of those who may profit from membership advantages. All who have an interest in concrete are eligible for membership. The grades of membership are described below.

Members have at hand in Institute publications the most complete fund of knowledge on concrete. The ACI Journal provides them with the latest information and ACI special publications provide them with the complete picture of specific problems. Through conventions, and regional and area meetings, they are afforded the opportunity of meeting those whose experiences provide the new information, and of exchanging ideas with them.

ACI's world-wide membership is growing in extent and participation —traveling a common road toward better, more economical and durable concrete structures. ACI provides a common ground in the search for and use of new "working tools" in concrete design, manufacture, and erection—and its interpretation.

Individual Members (U.S. and Possessions, Canada, Mexico, Central America, and West Indies)	**$20.00**
Individual Members (All other foreign countries)	**16.00**
Corporation Members	**65.00**
Junior Members—nonvoting (under 28)	**10.00**
Contributing Members	**135.00**
Student Members—nonvoting (under 28)	**5.00**

Please enclose remittance with application (cut here)

Board of Direction, American Concrete Institute **Date**_____
P. O. Box 4754, Redford Station
Detroit 19, Michigan

The undersigned hereby applies for admission to the American Concrete Institute as ☐ *Individual* ☐ *Corporation* ☐ *Contributing* ☐ *Junior* ☐ *Student Member and agrees to be governed by the Charter and Bylaws.*

Name and complete mail address of proposed membership (Address to which Journal is to be mailed—please letter)_____

_____

For Corporation Membership, ACI representative will be _____

Date of Birth (Juniors and Students only) _____
Year Month Day

College or University attending (Students only) _____

Signature_____

Month & Year of Graduation (Students Only)

(Proposed by) please print

For our records, please complete both sides of application.

NOTES on Membership Classification

The ACI advertising and editorial departments need definite information concerning job title, classification, business affiliation, and principal responsibility of all members. Thus, the information requested on the classification form below is especially important.

The applicant should designate his title or position such as: construction superintendent, research director, technician, draftsman, structural engineer, inspector, plant superintendent, highway engineer. The occupation of the applicant should be that which is **most frequently** performed. The principal responsibility of the applicant is that specific area of his job for which he is **primarily concerned.**

Classification is not based on what interests you, but on what you do—what office or job you fill. When completing the form below, please check the one item in each section that is most applicable to that particular section.

ACI editors want to know your interests—they welcome suggestions as to what you'd like to see discussed in the ACI Journal and of the possible sources. Please attach a separate note or letter.

MEMBER CLASSIFICATION

JOB TITLE _____

OCCUPATION. (Check the one most applicable)
☐ Arch ☐ Engr ☐ Construction Supervision ☐ Plant Management or Supervision ☐ Teaching ☐ Student ☐ Other (please state) .....

EMPLOYER _____
(Name of Company) (Street Address) (City and State, or Country)

☐ Architect ☐ Contractor ☐ Consulting Engr ☐ Engr Firm ☐ Manufacturer or Producer (specify product) .....
Government ☐ Fed ☐ State ☐ County ☐ City ☐ Educational Institution
☐ Commercial Testing Laboratory ☐ Public Utility ☐ Trade Assn ☐ Library
☐ Other (please state) .....

PRINCIPAL RESPONSIBILITY (Check the one most applicable)
☐ Design ☐ Construction ☐ Consulting ☐ Purchasing ☐ Sales ☐ Advertising ☐ Research ☐ Administrative (state position) .....
☐ Other (please state) .....

Do you ☐ Specify ☐ Authorize ☐ Recommend, purchase of materials or equipment?

Alpha Cement initiates weather forecasting service

A monthly weather forecasting service for cement users was recently initiated by Alpha Portland Cement Co., Easton, Pa. It is said to be the first specialized service of its type offered in the cement industry, and is available to all Alpha customers in the eastern half of the United States.

Purpose of the service, which provides more detailed weather information than that available from the U.S. Weather Bureau, is to help contractors, ready-mixed concrete companies, and concrete products producers plan their work for the coming month with better knowledge of what to expect from the weather.

The information contained in the monthly weather service is supplied to Alpha by Weather Trends, Inc., New York, a private firm of meteorologists specializing in long range forecasting. Each monthly forecast, printed in a 4-page bulletin, "The Alpha Weathervane," predicts temperature and precipitation expected for 1 month in advance. Specific climatological data and forecasts are also included for key cities. A detailed summary is included for each major region.

BRI announces program for fall conferences

The Building Research Institute will stage its 1960 fall conferences at the Shoreham Hotel, Washington, D.C., November 15-17.

The 3-day program will feature a research correlation conference on preassembled components in modern building, a session on structural foams, a conference-workshop on fasteners and anchorage devices for industrial curtain walls, and a conference on building research developed by the institute's research committee.

ACI member Rudolph C. Valore of Sika Chemical Corp., Passaic, N.J., is scheduled to discuss the various applications of foamed concrete in roof slabs, floor slabs, and walls at the session devoted to structural foams.

Open to the interested public as well as to BRI members and guests, the 3-day meeting is the third in the newly inaugurated series of BRI semiannual, multisubject conferences. Information regarding registration or further details about the conferences may be obtained from Harold Horowitz, assistant director for technical programs, Building Research Institute, 2101 Constitution Ave., Washington 25, D.C.

Bolles joins C-S-T staff

Ralph E. Bolles, formerly associated with Bechtel Corp., Los Angeles, has joined the C-S-T Engineering Co., Los Angeles.

Prentzas joins Leap associate

E. G. Prentzas has joined Leap Structural Concrete, Inc., Atlanta, Ga., as plant manager and chief engineer.

Admixtures for Concrete

• *By ACI Committee 212*

Concrete admixtures are classified into 11 groups: accelerators, retarders, air-entraining agents, gas-forming agents, cementitious materials, pozzolans, alkali-aggregate expansion inhibitors, damp-proofing and permeability-reducing agents, workability agents, grouting agents, and miscellaneous.

This 34-page report discusses each group and the important effects to be expected in using materials of each group.

Order from Publications Department, American Concrete Institute, P.O. Box 4754, Redford Station, Detroit 19, Mich.

Price 75¢

Honor Roll

Jan. 1-Aug. 31, 1960

Today ACI membership has reached an all-time high. Thousands of persons working in the concrete industry throughout the world need ACI services and ACI needs the best of their thinking. Tell your colleagues about ACI activities! Add your name to the Honor Roll next month.

Point System

1 point for Student; 2 points for Junior; 3 points for Individual; 4 points for Corporation; and 5 points for Contributing.

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Samuel Hobbs	32
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Walter H. Price	16
Roger H. Corbetta	15
Harry Ellsberg	15
Russell Porter	14
Joaquin Spinel L.	14
Antonio A. Henson B.	12
W. E. Moulton	12
C. P. Siess	12
Faraj Tajirian	10
J. Raymond Watson	10
Jacob J. Creskoff	9
Alfonso Golderos	9
James A. McCarthy	9
Gene M. Nordby	9
Geo. B. Southworth	9
Milton H. Zara	9
James Chinn	8
Martin J. Gutzwiller	8
H. C. Pfannkuche	8
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C. A. Willson	6
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Melton A. Croson	5
John E. Heer, Jr.	5
Adrian Pauw	5
Felix Colinas Villoslada	4
John G. Dempsey	4
G. Grenier	4
Wm. W. Karl	4
Joe W. Kelly	4
Narbey Khachaturian	4
Simon Lemar	4
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Jack Longworth	4
Willard A. Oberdick	4
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Michael Alexander	3
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J. E. Backstrom	3
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Evaluation of Strength Tests of Concrete

This ACI Bibliography (No. 2 in the series) lists and annotates 35 selected articles appearing in available technical publications issued from 1924 to 1958 and dealing with compression tests of concrete, variations in test results, and evaluation of tests. The articles explain the advantages and value of applying statistical methods in evaluating quality of concrete. In 8½x11-in. format, punched for insertion in three-ring binder. Price per copy is $2.00; $1.00 to ACI members.

Order from Publications Department, American Concrete Institute, P. O. Box 4754, Redford Station, Detroit 19, Michigan.

New Members

The Board of Direction approved applications in the following categories: 44 Individual, 2 Corporation, 8 Junior, and 16 Student, making a total of 70 new members. Considering losses due to deaths, resignations, and nonpayment of dues, the total membership now stands at 10,292.

INDIVIDUAL

BALDNER, ELROY O., Houston, Tex. (Struct. Engr.)
BAR, JAQUES, Montreal, Que., Canada (C. E.)
BIANCHI, PHILIP A., Newton, Mass. (Des.-Draftsman, Joseph L. Paley, Inc.)
BINDER, JOSEPH C., Milwaukee, Wis. (Vice-Pres., Milwaukee Fly Ash, Inc.)
BLAND, WYNSTON E., SR., Chattanooga, Tenn. (Pres., Wynston Bland Constr. Co., Inc.)
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BUNDS, PATRICK, Pleasanton, Calif. (Test. Engr., Pacific Cemt. & Agg., Inc.)
CRITCHFIELD, JUSTUS W., Fullerton, Calif. (Dep. Bldg. Insp., Twining Labs. of Sn. Calif., Inc.)
DELORD, CAMILLE, Bruxelles, Belgium (Admin. Dir. Gerant, Bur. SECO)
DOZIER, JOHN D., Louisville, Ky. (Struct. Engr., Girdler Constr. Corp.)
EYMAN, KRYSTIAN H., Warszawa, Poland (Prof., Tech. Univ. of Warsaw)
FAZIO, PASQUALE, Berkeley, Calif. (Job Engr.)
FLOOD, THOMAS CLENDON, Auckland, New Zealand (Asst. C. E., Auckland City Council)
GARRETT, GEORGE M., Austin, Tex. (Pav. Engr., PCA)
GIANGRECO, ELIO, Napoli, Italy (Prof., Univ. di Napoli)
HENSLEY, FLOYD E., Oklahoma City, Okla. (Brdg. Engr., Hudgins, Thompson, Ball & Assoc., Inc.)
INOUYE, RALPH S., Honolulu, Hawaii (Bldg. Supt., City & Cty. of Honolulu)
JACQUES, LUCIEN, Montreal, Que., Canada (Struct. Desr.)
JANICHE, WALTER, Rheinhausen, Germany (Dir., Hutten- und Bergwerke Rheinhausen AG)
JORGENSEN, FOLMER, Trorod pr. Vedbaek, Denmark (Dep. Mng. Dir., De danske Betonfabrikker A/S)
KELLY, ALBERT A., Seattle, Wash. (Cons. Engr.)
KOESTER, JOHN H., Austin, Tex. (Fld. Engr. Supv., PCA)
KUPPERMAN, MELVIN M., Chicago, Ill. (Struct. Engr., A. Epstein & Sons)
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MACINTIRE, WILLIAM H., Fayetteville, N. Y. (Engr.-Insp., O'Brien & Gere, Cons. Engrs.)
MINNER, WARREN A., Bakersfield, Calif. (Struct. Engr.)
MITCHELL, JOHN E., Wenatchee, Wash. (Brdg. Engr., Calif. Div. of Hwys.)
PADILLA, NARCISO L., Havana, Cuba (Struct. Engr., Saenz-Cancio-Martin-Gutierrez)
REZNICK, SIDNEY, Whitestone, N. Y. (Struct. Desr.)
RUST, WILLIAM DAVID, JR., Washington, D. C. (Struct. Engr., Pub. Bldgs. Ser.)
SCHNEIDER, GEORGE, Camden, N. J. (Struct. Engr., John G. Reutter & Assocs.)
SEN, RENUBHUSAN B., Calcutta, India (Supt. Engr., Govt. of W. Bengal)
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SMITH, H. L., Cincinnati, Ohio (Conc. Techn., Reading Central Mixed Conc., Inc.)
STALIS, HARRY, Brooklyn, N. Y. (Struct. Supv.)
SULLIVAN, JAMES B., JR., Bethesda, Md. (Struct. Engr., George M. Ewing Co., Archs. & Engrs.)
THORUD, BERT M., Phoenix, Ariz. (Arch.-Engr.)
THURLOE, R. L., Perth, Western Australia (C. E., Ready Mixed Conc. (W. A.) Pty. Ltd.)
UTSCH, LINUS F., JR., Allentown, Pa. (Sr. C. E., Air Prods., Inc.)
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ZACKRISON, H. B., SR., Arlington, Va. (Chf., Engrg. Div., Office Chf. of Engrs.)

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QUEBEC READY-MIX, INC., Quebec, Que., Canada (J. L. Bourret, P. Engr.)

JUNIOR

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FERNANDEZ-SEIN, LUIS, Hato Rey, P. R. (Engr.)
KASZAS, E. F., Calgary, Alta., Canada (Prod. Engr.)
MARQUOIT, LESLIE C., Berkeley, Calif. (Res. Engr., E. Bay Municipal Utility Dist.)
PORTERFIELD, BRUCE R., Murrysville, Pa. (Engr. in trng., Crucible Steel Co.)
SCHWEIZER, MARK, JR., Winter Park, Fla. (Struct. Engr., Nils M. Schweizer & Assoc., Arch & Engrs.)
SMITH, EUGENE FRANKLIN, Miami, Fla. (Struct. Desr.)

STUDENT

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BEER, ROBERT H., St. Louis, Mo. (Washington Univ.)
CUNNINGHAM, LYLE R., Phoenix, Ariz. (Ariz. State Univ.)
GEBHARDT, JORGE, Guatemala, Guatemala (Univ. de San Carlos)
GRAHAM, WILLIAM G., Sept Iles, Que., Canada (St. Patrick's College)
HEITHECKER, GARY LEE, Grapevine, Tex. (Sn. Methodist Univ.)
KUDSI, MOHAMMED TAKIDDIN, W. Lafayette, Ind. (Purdue Univ.)
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OPLE, FELICISIMO S., JR., Bethlehem, Pa. (Lehigh Univ.)
ROBLES BIDOT, JAIME, Rio Piedras, P. R. (Univ. of P. R.)
SANDUSKY, R. ROBERT, Kansas City, Mo. (Finlay Engrg. Coll.)
THOFT-CHRISTENSEN, P., Lyngby, Denmark (Techn. Univ. of Denmark)
TREFOGLI A., GUILLERMO, Lima, Peru (Univ. Nacl. de Ingenieria)
VARELA, HECTOR M., Mayaguez, P. R. (Univ. of P. R.)

Tools, Materials, Services

Under this heading note is made of producer literature and products of presumed technical interest to ACI users of tools, equipment, materials, accessories, and special services.

Adhesive coating for masonry walls

Adfinish, adhesive coating for finishing exterior or interior masonry walls, beautifies as it fills and protects according to the developer.

The coating comes in a smooth or textured finish and can be applied either by brush or spray to dry, damp, or painted surfaces. Adfinish is available in white, yellow, and gray. The product is self curing, and once applied, finishing is complete. — Swift and Co., 4115 Packers Ave., Chicago 9, Ill.

Joint sealing compound

Horn-Tite, rubber asphalt joint sealing compound, has recently been introduced for sealing joints and cracks in concrete highways, drives, streets, bridges, and parking areas.

The compound is available in two forms: Hot-Poured and Cold-Applied. The first type is specifically for horizontal joints to prevent the infiltration of water, soil, or other solid materials into the joint spaces.

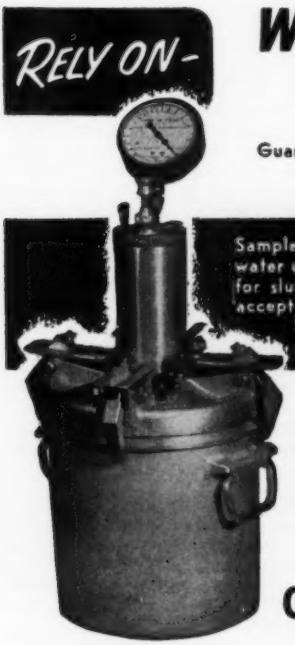
Manufacturer states that compound, unlike bituminous materials, will not flow or extrude in summer, or crack and pull away from joint walls in winter. Also, it is said to give lasting sealing effectiveness in both hot and cold weather and remains intact for years. — A. C. Horn Companies, Division of Sun Chemical Corp., 2133 85th St., North Bergen, N.J.

Compact turbidimeter

Soiltest has designed a compact new model of the Wagner turbidimeter for determining the fineness of portland cement.

This model, CT-365, includes all the apparatus needed in testing to determine the fineness of portland cement as represented by specific surface expressed as total surface area in sq m per g of cement.

The test apparatus consists of a source of light of constant intensity, adjusted so that approximately parallel rays pass through a suspension of sample cement and impinge on the sensitive plate of a photoelectric cell. The current generated by the cell is measured with a microammeter and the indicated



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Simplified device for testing light weight concrete. Precision instrument made of solid bronze. Requires no computation — no special training. Reliable — durable.

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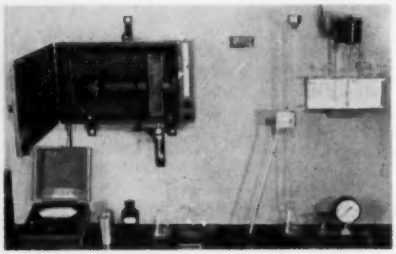


reading gives a measure of the suspended cement's turbidity.

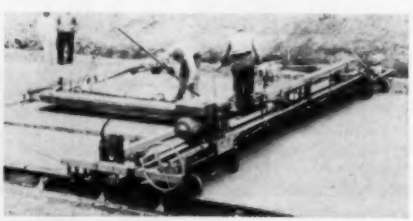
Included in the unit are photoelectric cell and light source in a metal cabinet, a timing burette and stand, a microammeter, a wet screening assembly, sedimentation tank, flasks, test tubes, and a stirring apparatus.—Soiltest, Inc., 4711 W. North Ave., Chicago, Ill.

Forming the joints in concrete pavement

An original method for forming contraction joints in pavements eliminates hand-forming or sawing of joints and obtains the necessary plane of weakness by a joint former inserted directly into the fresh concrete after placing and machine finishing. The insert, called Unitube, is a hollow strip of lightweight galvanized steel. The top is $\frac{3}{8}$ in. wide. The $2\frac{1}{4}$-in. sides taper together at the bottom to permit easy insertion into the fresh concrete. Each of these flanges turns outward and upward at the bottom to form a claw, imbedding each side of the flexible metal insert to a slab so that it spreads when cracking occurs.

The folded metal strips are produced in 2 and 3 in. depths and are pre-cut to desired lengths. Ten-ft lengths, which interlock end-to-end, are installed to form the longitudinal joint. Transverse joints are placed from the side forms to the center line where they butt against the longitudinal joint.

A two-man crew works from a specially designed rig which rides the forms behind the contractor's paving train. This machine provides guides for accurate placement of the metal insert and a vibrating "T-bar" device is used to form a groove in the fresh concrete and then push the insert to the proper depth. The $\frac{3}{8}$ in. wide flat top is left flush with the finished surface.



Installing metal insert to form longitudinal and transverse joints. Workman on the forward platform is inserting the joint insert through guide slots. Transverse inserts are installed from the rear platform at the same time.

After the pavement has cured, a one-man operated mechanical vibrating machine called a "crimper" is run down each joint, folding the top of the metal tube inward and downward. This produces a joint $\frac{1}{2}$ in. deep to carry the sealing compound.—Middlestadt Corp., Baltimore, Md.

After the concrete is cured tre metal inserts are crimped with a special vibrating machine

Literature Available

Pertinent details on the latest equipment and products on the market are available in recently released literature. Exact titles of the booklets and catalogs are indicated in capital letters. They may be requested directly from the manufacturers listed below.

ATLAS COMPO FORMS—New Atlas Compo forms are described. —Irvington Form & Tank Corp., 100 William St., New York 38, New York.

HANDBOOK OF CRUSHING — New edition of Pennsylvania Crusher's handbook. The 40-page book has been prepared to provide a better understanding of the entire subject of mechanical size reduction. Various reduction methods are described and discussed, as are the many kinds of crushing equipment. Amply illustrated with diagrams and charts for easy understanding.—Pennsylvania Crusher Division, West Chester, Pa.

SEALTIGHT PRODUCTS FOR CONCRETE CONSTRUCTION (Catalog No. 100)—Catalog covers the line of products available for use in the construction of highways, bridges, airports, and concrete construction of all types. It includes expansion joints, joint sealing compounds, control joints, sewer joint compounds, waterstops, and curing compounds.—W. R. Meadows, Inc., 26 Kimball St., Elgin, Illinois.

X-RAY LITERATURE LIST AVAILABLE — List of references covering 1930 to 1960, giving authors and publication names, article tities and publication dates for 376 papers on x-ray analysis subjects. — Philips Electronic Instruments, 750 South Fulton Ave., Mount Vernon, N.Y.

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